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ASTEROID ORBIT DETERMINATION AND ROTATIONAL PERIOD CALCULATION WITH CCD ASTRONOMY

by

DANIEL C. BURTZ

B.S., United States Air Force Academy, 1997

A thesis submitted to the Graduate Faculty of the

University of Colorado at Colorado Springs

in partial fulfillment of the

requirements for the degree of

Master of Engineering

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Asteroid Orbit Determination and Rotational Period Calculation with CCD Astronomy

Thesis directed by Professor Charles Fosha, Ph.D.

This paper presents data collected and analyzed relating to photometry and astrometry of asteroids. All observations were accomplished at the U.S. Air Force Academy Observatory. The photometry involves determining the rotational period of asteroid 583 Klotilde. Astrometry was performed on asteroid 1035 Amata and the calculated position was used to determine its orbital elements.

Klotilde was selected for rotational period determination based on its relatively low magnitude, favorable viewing position, and no previous rotational period information. Two hundred six images of Klotilde were taken and analyzed over four viewing nights. A Photometrics (PM512) Charge Couple Device (CCD) camera attached to a 61-cm Cassegrain telescope was used for these observations. Using NOAO's IRAF software, the magnitudes of Klotilde and several comparison stars were determined. Using an Excel spreadsheet, differential photometry was performed and the lightcurve was plotted. The four nights of data gave a 9.210 ± 0.005 hour synodic period with an amplitude of 0.18 magnitudes.

Thirty-two images of Amata were taken on six different viewing nights. The images were taken with an ST-8 CCD attached to a 41-cm Cassegrain telescope. The data was reduced with the Astrometrica software package, which calculated the right ascension (RA), declination (Dec), and magnitude of Amata using several comparison stars. The computed RA and Dec, along with the times of observation were then used to determine the orbital elements of the asteroid.

Gauss' method of angles only orbit determination was used to calculate the orbital elements. Three separate computer programs were used to calculate the orbital elements. The programs used were the Gauss-Encke-Merton (GEM) program by J.M.A. Danby, ORBDET by Montenbruck and Pfleger, and FIND_ORB by Bill Gray. The closest match to the published orbital elements came form FIND_ORB which used a batch least squares approach to use all of the observational data. ORBDET and GEM used only three observations at a time and yielded less accurate results. Due to suspected errors in the GEM program, it was the least effective method used. The validity of our observations was verified by the Minor Planet Center, as well as FIND_ORB and ORBDET.

ACKNOWLEDGEMENTS

I have several people I would like to thank for their assistance with this thesis. I would first like to thank the Department of Physics at the U.S. Air Force Academy for allowing me the use of the observatory and it resources in collecting my observations. Assistance in operating the observatory equipment was provided by Jesse Thompkins and additional observations of 1035 Amata were provided by the Physics 480 class taught by Maj. Charles Wetterer.

I would also like to thank the members of my defense committee, Charles Fosha, Ph.D., Douglas Kirkpatrick, Ph.D., and Gary Yale, Ph.D., for their inputs and suggestions. Dr. Fosha also served as my thesis advisor.

The Air Force Institute of Technology defrayed many of the software and related text costs.

I would especially like to thank the two people who worked most closely with me on my research. Roger Mansfield assisted with the orbit determination. He suggested the programs to use and assisted in modifying the GEM program. He also explained the mathematical techniques employed by the Gauss' angles only method of orbit determination in addition to reviewing my thesis and providing comments.

Charles J. Wetterer, Ph.D., Department of Physics, USAFA, assisted in collecting and analyzing the observational data. I also used an Excel spreadsheet written by him to perform differential photometry. He also co-authored the rotational period paper submitted for publication in the Minor Planet Bulletin and reviewed my thesis to make sure I accurately described all the observational and data reduction techniques employed.

Both of these gentlemen invested a great deal of time and energy into helping me with my research and writing for which I am very grateful.

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CHAPTER I

INTRODUCTION

Astronomy Background

Our solar system contains thousands of objects commonly referred to as asteroids or minor planets. For the most part, they are just rocks in space that orbit around the Sun. There is much theory on the origin of asteroids, from the break-up of planets, to pre-stellar matter that failed to form into a separate planet. Asteroids can be found throughout the solar system in a variety of orbits. The vast majority are located in fairly circular orbits located between the orbits of Mars and Jupiter, an area often referred to as the main asteroid belt. Other asteroids of particular concern have orbits that intersect that of the Earth, which are referred to as near earth asteroids. Almost all asteroids are relatively small with diameters of less than five miles although four or five dozen may exceed one hundred miles.[1]

Throughout this thesis, references will be made to an object's magnitude. Magnitudes are an astronomer's method of indicating brightness. The magnitude scale is an inverse scale with lower numbers indicating brighter objects. Before the advent of the telescope, all observations of the night sky were obviously made with only the naked eye. Early astronomers classified all visible stars on a scale of 1 to 6 based on their apparent brightness. Since all objects are different distances from the Earth and either reflect or generate different amounts of energy; the apparent magnitude is not an accurate estimation of its true luminosity. For some observational work astronomers correct apparent magnitudes to absolute magnitudes. The absolute magnitude is the apparent magnitude a star would have if it were at a distance of 10 parsecs or 32.6 light years from the Earth. All of the magnitudes discussed in this thesis will refer to the apparent magnitudes measured by our instruments.

This thesis contains two distinct sections. The first section deals with the photometry of an asteroid to find its magnitude. By plotting this magnitude as a function of time, a rotational period for the asteroid can be determined. The second section describes the astrometry of a different asteroid to determine its right ascension and declination. Using these coordinate positions and the observation times, Gauss' method of angles only orbit determination was used to calculate the orbital elements of the asteroid. The specifics of the data collection procedures and data analysis will be included in their respective chapters.

Observational Equipment

All observations were taken at the U.S. Air Force Academy Observatory, which can be seen in Fig. 1.1. The location of the observatory is 104° 52' 51.9" W, 39° 00' 25.3" N, at an altitude of 2180m. A large portion of observational astronomy is conducted with Charge Couple Devices (CCDs). These cameras collect photons on an electronic chip during an exposure. After the shutter closes an electric potential is used to "roll" the exposed pixels off the chip so that they can be counted. CCD astronomy is considered visual astronomy like naked eye observations because it makes observations in the visible portion of the spectrum. Infrared and radio are two types of astronomy that observe in other than the visible spectrum.

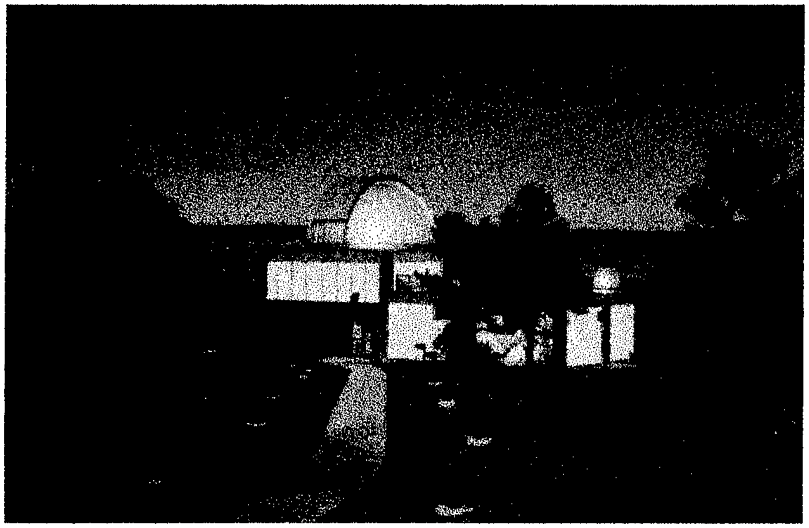

Fig. 1.1 USAF Academy Observatory

CCD photometry was accomplished with a Photometrics (PM512) camera attached to a 61-cm Cassegrain telescope with an equatorial mount. CCD astrometry was accomplished with a 41-cm Cassegrain telescope with an equatorial fork mount. The Cassegrain configuration indicates that the telescope achieves a longer focal length by means of two interior mirrors. An illustration of a Cassegrain configuration can be found in Fig. 1.2. An equatorial mount aligns the prime axis of the telescope with the Celestial North Pole. Therefore, to track an object the telescope only needs to rotate in one axis. A fork mount resembles a tuning fork. Depending on the instrument sizes, there can be problems viewing objects near the Celestial North Pole with a fork mount. Pictures of the 61-cm and 41-cm telescopes can be seen in Fig. 1.3 and 1.4 respectively.

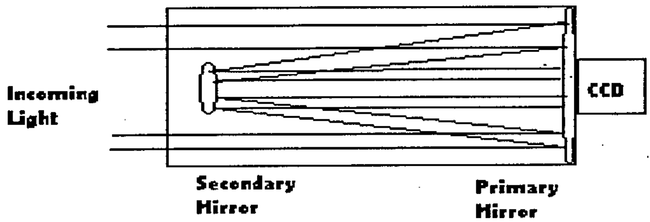

Fig. 1.2 Cassegrain Configuration

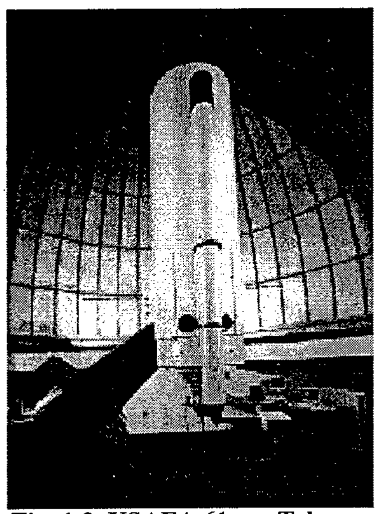

Fig. 1.3 USAFA 61-cm Telescope

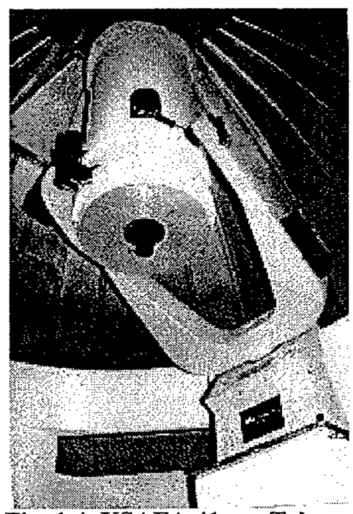

Fig. 1.4 USAFA 41-cm Telescope

Photometry vs. Astrometry

The basic principle of photometry and astrometry are the same, but their end goals differ. Both processes use a CCD to collect visible light photons. For asteroids, the light is generated by the Sun and reflected off the asteroid. Neighboring stars obviously generate their own light. The neighboring stars will hereafter be referred to as comparison or reference stars. After an exposure is taken the photons are electronically counted and a computer-generated representation of the objects in the field of view of the camera is displayed.

In photometry, only the number of photons and therefore the magnitudes of the asteroid and comparison stars are calculated. The data are then differentially corrected and plotted. The astrometry software used also calculates the magnitude, but this is only a secondary calculation as far as our research was concerned. Our main purpose was to calculate the position of the asteroid. Since all stars can be considered fixed in space, the astrometry software can use the known coordinates of several of the comparison stars to determine the coordinates of the asteroid in question.

The calculated positions are given in units of right ascension and declination. Right ascension and declination are the same for any object regardless of viewing time or viewing location. This feature makes them the units of choice for astronomers who can use the coordinates to determine when a particular object is visible from their observing location if it is at all.

CHAPTER II

583 KLOTILDE PHOTOMETRY OBSERVATION PROCEDURES

Selection of 583 Klotilde

The purpose of the photometry research was to select an asteroid to observe whose rotational period had not previously been determined. To begin, Guide software version 3.0 was used to identify asteroids that would be visible from our location. The Guide software incorporates data from the Hubble Guide Star Catalog.[2] Although these programs have different names, they are quite different. In this thesis, Guide will refer to the program used to see the positions of asteroids and stars at different viewing times. The Hubble Guide Star Catalog is a database of celestial objects used by Guide as well as other programs mentioned later. Guide allowed us to input the viewing time and location so we could see which asteroids were visible during our observation time. Obviously the asteroids we were considering had already been discovered but due to the large number of asteroids in our solar system, several of them had not been observed extensively. The Guide Star Catalog also contains the magnitudes for the asteroids we were considering. Since a higher magnitude star is dimmer, we arbitrarily set an upper magnitude limit of about 19, which would be close to the limit at which our telescope could detect the asteroid.

From a list of approximately five candidate stars we consulted information by Harris to see which of those asteroids did not already have a rotational period determined.[3] Based on this information, asteroid 583 Klotilde was selected. Klotilde was discovered on 1898 Dec 31 in Vienna by J. Palisa.[4] Its approximate diameter can be estimated at about 86 km based on its magnitude; however, no guess as to its shape can be made.[5]

Planning the Observations

Once 538 Klotilde was selected, we needed to plan the observations. Again consulting Guide, we were able to adjust the time settings for the night we planned to observe. The software showed us the asteroid with respect to the other stars that would be in the field of view during that time. Since the CCD has a field of view of only a few arcminutes, it was important to ensure that there was at least one suitable comparison star in the field of view with the asteroid. A suitable comparison star should be relatively bright with a magnitude of approximately 10 to 17. One comparison star is essential; however, two or three are desirable as will be seen later in the discussion of data reduction. If there weren't any suitable comparison stars on a particular night then changing observing nights was considered.

Another factor to consider is that over the course of the night the asteroid will be moving. By changing the settings in Guide to several hours later, we could see the general direction and distance it would travel. It was necessary to make sure that the asteroid didn't travel too far away from the comparison stars so that they would no longer be in the same field of view. We also checked to make sure the asteroid would not pass too close to another star which would interfere with determining which photons came from which object. All of these considerations needed to be made along with the weather forecast. Observing on consecutive nights would make determining the rotational period easier, particularly for asteroids with longer periods. Therefore selecting an observing night was a balance between favorable comparison stars on consecutive nights and the weather. Invariably something unexpected will occur while observing; therefore, backup nights are always a good idea.

Observing Klotilde

A 61-cm Cassegrain telescope was used for all photometry observations. Prior to observations each night, the CCD needed to be cooled with liquid nitrogen. A temperature of -120° C was desired for observations. The telescope was also focused to give a clear image. All of the images taken of 583 Klotilde had a five-minute exposure time using a standard Johnson R band (red) filter. An exposure of this length makes any error due to shutter inefficiency negligible. The images were taken using PMIS software. Four nights of observations were taken of Klotilde. See table 2.1 for a list of the observing nights, the time of observations, number of observations, and coordinates of the asteroid that night. Note that the right ascension and declination refer to the telescope pointing coordinates not the coordinates for Klotilde specifically. Since the asteroid is orbiting the Sun and not fixed in space like stars are, these pointing coordinates are actually changing continuously over the four or five hours that observations were made. Over these intervals however, the changes are not particularly relevant.

Date	Start time	Stop time	# of Obs	RA J 1998	Dec J1998
UT 98 Feb 13	UT 4.8	UT 10.3	48	8 53 76.9	5 49 46
UT 98 Feb 22	UT 3.4	UT 9.0	52	8 46 36.7	6 16 30
UT 98 Mar 12	UT 1.9	UT 9.4	56	8 38 34.0	7 07 16
UT 98 Mar 13	UT 2.8	UT 7.5	50	8 38 06.1	7 12 33

Table 2.1. Photometry Observing Log

With each observation, an observing log was filled out with other procedural information. The logs from all observations at the Air Force Academy Observatory are maintained in a binder for possible future reference. Included in the log is the start time, read to the nearest second (Universal Time) from a wristwatch synchronized with the U.S. Naval Observatory master clock. The exposure duration is specified with the camera software. The pointing coordinates of the telescope during the exposure are also recorded. Although the telescope control software automatically tracks the same position in the sky correcting for the Earth's rotation, small offset adjustments are needed periodically. In the event that the telescope needs to be moved for something such as viewing another object or refilling the liquid nitrogen dewar it could be repositioned later. The filter used was also recorded. For some astronomy purposes different filters are used for the same image, but as previously mentioned, all of our exposures used the R filter.

Other useful information recorded in the log is the hour angle, which indicates distance west (positive) or east (negative) from the meridian passing through the Celestial North Pole and the local zenith position. It is not used directly in data analysis, but can be useful in determining how late observations can be made. Also recorded was the airmass, which indicates the amount of atmosphere through which the telescope is looking. Looking directly overhead, the telescope is looking through one airmass. For angles off zenith, the amount of atmosphere the telescope is looking through increases. The airmass also increases accordingly. As with the hour angle, the airmass is not used directly in data analysis but can be useful as a reference. If an image is blurred or hazy the airmass can be checked to see how close the object was to the horizon at the time of the exposure. Light from objects closer to the horizon has more atmosphere to pass through and can be more distorted.

In addition to the actual images of Klotilde approximately five flat field images and ten bias frames were taken each night. Flat field corrections take into account the pixel-to-pixel sensitivity as well as other unavoidable effects such as vignetting or dust on the CCD.[6] The flat field images were 25-second exposures taken either of the illuminated inside of the dome or the twilight sky. Since new flat field images are needed each night observations are made, both of these techniques were used depending on the time of observations. The bias frames are zero second dark current exposures. A similar exposure called a dark frame (discussed in the astrometry section, Chapter V) was not needed since the CCD was cooled to less than –100° C. The purpose of these additional frames will be discussed along with data reduction.

CHAPTER III

583 KLOTILDE DATA REDUCTION

Data Reduction Software

The PMIS software saves the CCD images in the Flexible Image Transport System (FITS) format[7], which is the accepted format for professional astronomers.[6] These files are then transferred to a Sun computer workstation where the data reduction is actually accomplished. The software used to reduce the images is DIGIPHOT [8, 9], which is also the standard among professional astronomers.[6] Specifically we used aperture photometry, which is a package in the DIGIPHOT software. This software runs on the Image Reduction and Analysis Facility (IRAF) package available through the National Optical Astronomy Observatories (NOAO).

Image Corrections and Analysis

After importing the FITS images and organizing them into user specified directories, they must be converted to the proper image format used by IRAF. The bias and flat field images as well as the light images are all converted to this format. Next, a master bias image is created from an average of all the bias frames. IRAF has a command that allows the user to define the parameters and then it automatically performs this operation. This master bias is then subtracted from each of the light images as well as the flat field images. A function similar to the one used to create the master bias image is used to create a master flat field image. It is then necessary to normalize the average pixel value in the master flat field to 1 before applying the flat field correction into each of the light images. The light images are now ready to be displayed for analysis.

A picture of the image is displayed on the screen. After viewing a few images from the beginning and end of the night it is possible to determine which stars will be used as comparison stars. The comparison stars should be similar in magnitude (visual inspection is sufficient), and should appear in all or most of the frames. It is also important to select comparison stars that are not too close to the edges of the frame since this can affect the pixel count. For our purposes, one to three comparison stars were selected for each observing night. An example of a raw image from the PMIS software can be seen in Fig. 3.1. Notice the donut shaped smudges in parts of the image. These smudges are removed by applying the flat field correction. Fig. 3.2 contains the same image, which has been enhanced to simulate the bias and flat field corrections by adjusting the contrast scale. Klotilde is the object indicated by the yellow arrow. The two other stars identified with red arrows were used as comparison stars. The bright star towards the top indicated by the blue arrow could also have been used, but the reason it was not will be discussed later.

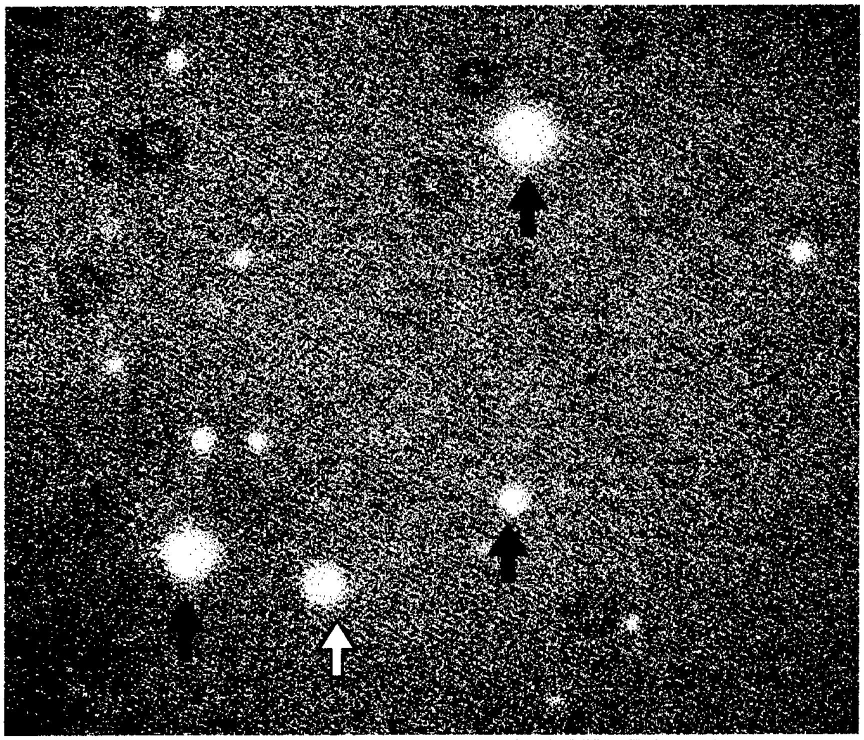

Fig. 3.1 Raw PMIS 583 Klotilde Image. FOV: 3.7 X 3.7 Arcminutes.

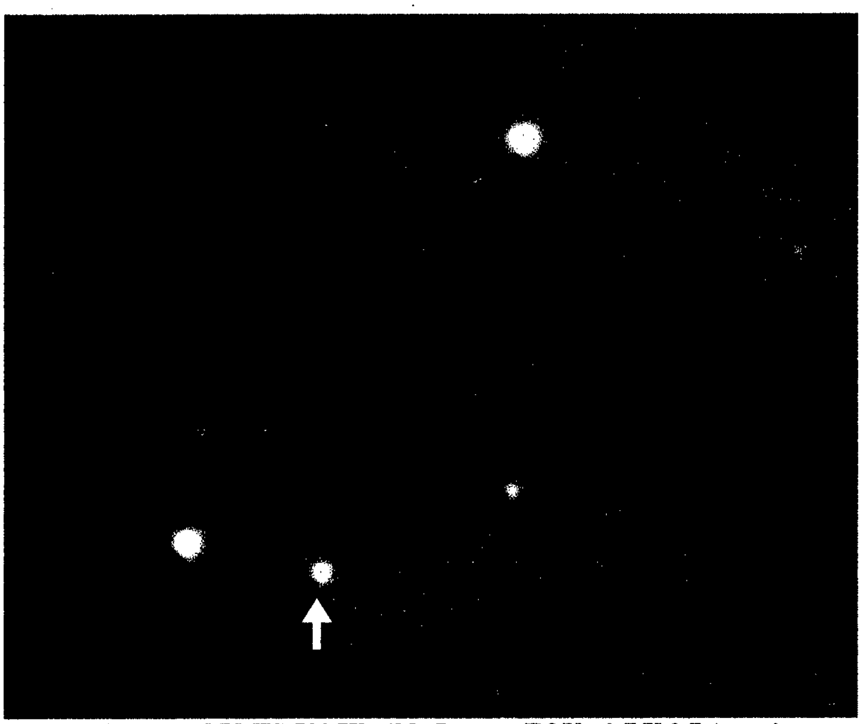

Fig. 3.2 Enhanced PMIS 583 Klotilde Image. FOV: 3.7 X 3.7 Arcminutes

The images then need to be inspected for cosmic rays. One of the disadvantages of CCD photometry with long exposures is that it is susceptible to pollution from cosmic or galactic rays. The cosmic rays usually appear as highly concentrated bright pixels, which stand out from the surrounding dark background. Some cosmic ray spikes can be seen as single bright pixels in Fig 3.2. The stars and asteroids are quite large compared to the cosmic ray spikes making them easy to identify. Only those spikes in close proximity to the asteroid and comparison stars are of concern. Close proximity is difficult to define but a sense of this is developed after processing a few images. If the spikes are located extremely close to the star or asteroid however, it can be difficult to distinguish the spike from the object's photons. Since the cosmic rays will increase the photon count for the object they are close to, it is necessary to eliminate them from the image. The DIGIPHOT software allows the user to replace the pixels affected by the cosmic rays with the average value of the surrounding pixels. This is only an approximation, but it yields a pixel value much more realistic than the cosmic ray value.

After the cosmic ray spikes of concern have been eliminated, the user identifies the center of the object desired with the mouse and an aperture around the object is specified to count the photons in. DIGIPHOT then counts the total number of photons within the specified aperture. Usually several aperture sizes are selected and the best one is selected later. A plot of the photons is then displayed. This is inspected to ensure no remaining cosmic ray spikes within the specified apertures are corrupting the photon count. The cosmic rays will lie outside the main photon curve since they have a much higher intensity than the surrounding pixels. If additional cosmic ray spikes are detected they need to be eliminated as discussed previously and the photometry process is repeated.

In each frame, photometry needs to be accomplished for the asteroid and each comparison star. It is important to select the asteroid and comparison stars in the same order for each frame. The result of this process is a file, which includes the calculated magnitude of each object selected, and the error for each calculation. The magnitude and error based on each aperture specified is calculated. Usually the calculations from a larger aperture are used unless the selected object is too close to the edge of the frame. In this case, a smaller aperture size may be used. A little experimentation provides the proper balance of including all the photons in the aperture while still remaining in the frame boundaries.

From this file, the magnitude and error for each object are recorded in a notebook. This data is then used to perform the differential photometry analysis to determine the rotational period.

CHAPTER IV

583 KLOTILDE DATA ROTATIONAL PERIOD DETERMINATION

Lightcurve Information

The lightcurve referred to in this thesis is a plot of the magnitude versus time. The purpose for plotting the lightcurve of an asteroid is based on the fact that asteroids have irregular shapes. For the purpose of illustration assume that an asteroid has a roughly potato shape. As the asteroid rotates, different faces or sides of it face the Earth. When an edge with a larger cross sectional area faces the Earth, it will reflect more light than an end with a smaller cross sectional area. When more photons are reflected towards the Earth the CCD registers a higher photon count and thus a lower (brighter) magnitude is determined for that asteroid. As the asteroid makes one complete rotation, the sides facing the Earth will be edge, end, edge, end.

The potato asteroid example assumes that the two edges, or put another way, the front and the back, have roughly the same cross sectional area, as do the two ends. Under these conditions, a plot of the magnitudes should reveal a figure similar to a sine wave with two maxima and two minima. By matching up the location where the curve begins to repeat itself, the rotational period can be determined. If the rotational period is such that most of a complete rotation can be observed in one or two viewing nights it should be possible to determine the rotational period. In Fig. 4.1, the period (where the curve repeats itself) is the entire length of the frame,

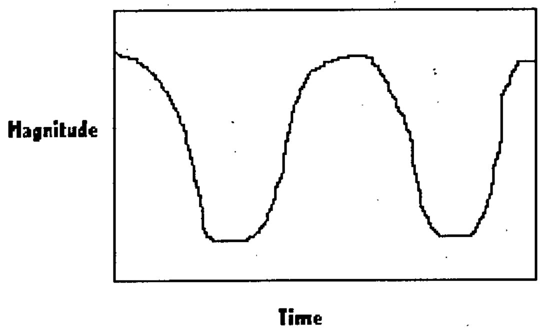

Fig. 4.1 Potato Shaped Asteroid Lightcurve

Of course in reality, most asteroids are not as symmetric as the potato example. If one edge or end has a slightly larger cross sectional area than the other does, then the light reflected and therefore the respective magnitudes will be different. In this case, one of the maximums or minimums might be larger than the other, as shown in Fig. 4.2. However, in most circumstances it is still reasonable to assume that a double maxima and minima lightcurve will result.

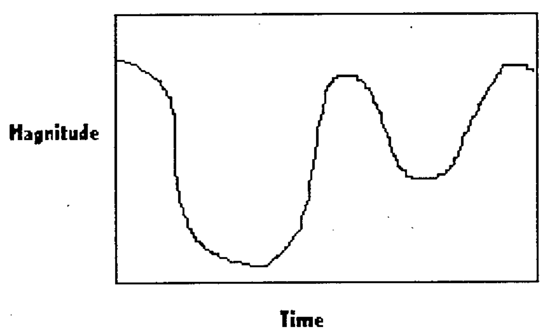

Fig. 4.2 Irregularly Shaped Asteroid Lightcurve

Under certain circumstances, it is impossible to determine a rotational period with any certainty. If the asteroid should happen to be roughly spherical like a planet, (improbable since many asteroids are loose conglomerations of smaller chunks and rocks, or the result of catastrophic collisions) then relatively no change would be detected in the plotted magnitude making a rotational period determination extremely difficult. Also, if the asteroid is rotating extremely slowly, then several nights of observations are necessary just to observe a fraction of the period. This would make it a poor candidate for observation. The main reason for this is that several consecutive nights of observations would be necessary which is complicated by trying to find consecutively clear nights. After plotting one night's data, it would be possible to conclude whether or not this is a good asteroid for further observations. The results from such a scenario could still be useful however, by simply stating that the named asteroid has a rotational period of at least X hours. This could be of benefit to a future astronomer who is trying to find an asteroid with a relatively short period.

Differential Photometry Purpose and Calculations

The magnitude and error data recorded from the DIGIPHOT calculations were then fed into an Excel worksheet. The worksheet for each observing night can be found in Appendix A. The format for this worksheet was written by Maj. Charles Wetterer from the Air Force Academy Department of Physics for use in differential photometry. Differential photometry is the process of determining a difference in magnitudes based on observational calculations. The reason this process is needed is that the observations are made over a long period of time (four or five hours for our data) for each observing night. Over this period, it is likely that clouds will drift in and out of the field of view of the telescope, which will reduce the number of photons reaching the CCD. There is also a change in the airmass over the viewing period, which affects the number of photons collected. Differential photometry considers these factors and adjusts the calculated magnitude accordingly.

The differential corrections are applied to each object (asteroid and comparison stars) against each other object. First, the magnitude of each comparison star is subtracted from the asteroid

$$\mathbf{m_K - m_1} \tag{4.1}$$

$$\mathbf{m_K - m_2 - (m_1 - m_2)} \tag{4.2}$$

$$\mathbf{m_K - m_3 - (m_1 - m_3)} \tag{4.3}$$

where **K** represents Klotilde and the numbers represent the comparison stars

The third term subtracted from equations 4.2 and 4.3 is used to adjust the values to the first comparison star. This term is actually the average of all of the individual observations. The time is computed at the middle of the exposure time as

$$\mathbf{Hour + (Min / 60) + (Sec / 3600) + (Exposure\ length / (3600 * 2))} \tag{4.4}$$

An offset is applied to correct for the change in observed magnitudes over the entire night. The offset is the magnitude of the first comparison star, which is taken from the Guide Star Catalog. The offset is added to equation 4.1 to get the combined magnitude.

$$\mathbf{m_{comb} = (m_K - m_1) + offset} \tag{4.5}$$

The combined error is the square root of the sum of the squares of the individual errors, which were calculated by DIGIPHOT.

$$\sigma_{comb} = \text{Sqrt}\,(\sigma^2_K + \sigma^2_1) \tag{4.6}$$

The combined magnitude of the asteroid with the offset correction is then plotted versus time to obtain the lightcurve.

Check for Variable Stars

The magnitude of the comparison stars are also corrected against each other and plotted to determine if any of them are variable. An example of such a plot is shown in Fig. 4.3. The plot should be roughly a horizontal line. Should there be fluctuations in the magnitude, it is possible that one of the stars is variable. A variable star is one whose light output varies due to a variety of reasons that will not be discussed here.

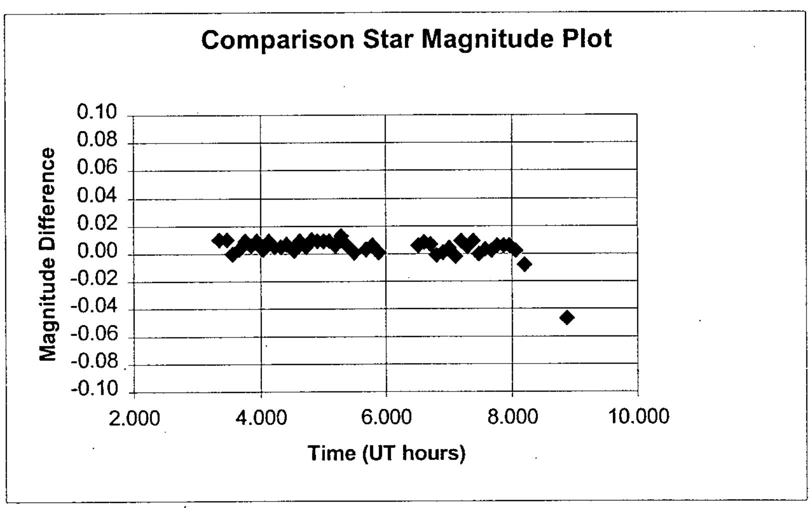

Fig. 4.3 Comparison Star Magnitude Plot

It is interesting to note that the data from one night's observations indicated that one of the comparison stars was indeed a variable star. This is the star in the upper part of Fig. 3.2 indicated by the blue arrow. See Fig. 4.4 for the comparison star magnitude plot for that star. The star was checked against a published list of variable stars, but was not found on it. With the suspicion of being variable, that star was not used as a comparison star for differential photometry. Luckily, there were two other candidates for comparison stars from that night's data. Determining the exact nature of the variability of that star provides and excellent opportunity for further astronomy research. This star's designation is GSC 223:1761 and is located in the constellation Cancer.

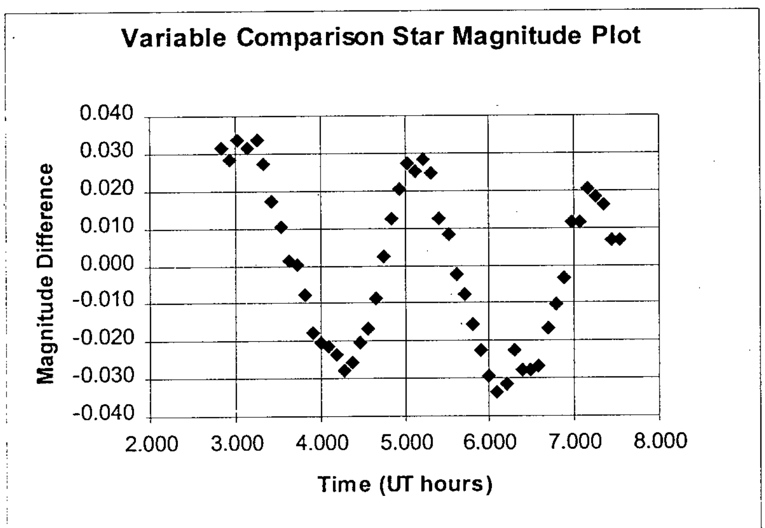

Fig. 4.4 Variable Comparison Star Magnitude Plot

Analyzing Klotilde's Lightcurve

After the differential corrections were made, the magnitude of the asteroid was plotted versus time. The data from the first night indicated what appeared to be a double minima and a double maxima. We were concerned however, that the amplitude was only about 0.18 magnitudes. In addition, the comparison star plot did not resemble what we expected it to look like. We decided to attempt another night of observations on Klotilde to see if new data could confirm that the data from the first night was reliable. The second night's data indicated a very distinct and smooth curve varying only towards the end of the evening as Klotilde was moving close to the horizon. This data also produced a double minima and double maxima lightcurve. The comparison star plot was also much more acceptable.

With this information, it appeared as if Klotilde possessed a period of approximately 4.5 hours. This was difficult to state with certainty, however, since more than a week had passed between the two nights of observations. This allowed for other integer multiples of the period based on the number of aliases. The number of aliases results from the frequency of our sampling against the entire period. If the observation rate is too small then the period indicated by our data may not cover one entire period. Therefore, two more consecutive nights of observations were conducted. Observing on consecutive nights reduces the number of possible aliases.

The third night of observations was our longest observing night. The data from that night indicated a triple minima and maxima lightcurve. The period associated with this lightcurve was approximately 9.2 hours. A triple minima and maxima lightcurve is unusual, which made us suspicious because an asteroid with such a lightcurve would have a particularly unusual shape. Adding the data from all four nights confirmed our data from the third night that the lightcurve did contain a triple minima and maxima. By adjusting the period offset, we were able to find the period by visually determining when the curves from each night best lined up. The most accurate period we could determine from our data was 9.210 ± 0.005 hours. This is a synodic not sidereal period. A sidereal period is the time it takes for the asteroid to return to a reference position in inertial space. The synodic period is the time for the same part to the asteroid to be facing the Earth. Since we are observing from the Earth, this is the type of period we are able to calculate. The synodic period may be either shorter or longer than the sidereal period depending on the direction of rotation of the asteroid and its position in the solar system relative to the Earth. The amplitude without error considered was determined to be 0.18 magnitudes. A graph of the final lightcurve can be found in Fig. 4.5.

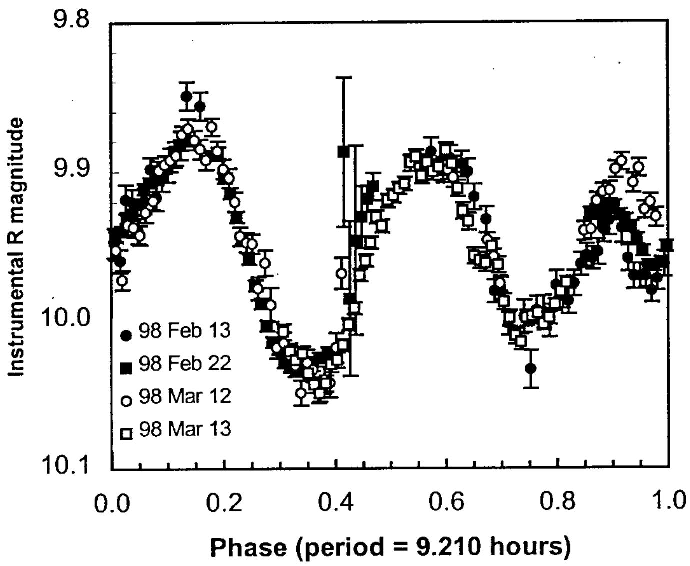

Fig. 4.5 Final Lightcurve Plot for 583 Klotilde

Results Submitted for Publication

Since the rotational period of 583 Klotilde had not previously been recorded, we submitted an article summarizing our findings to the Association of Lunar and Planetary Observers for publication in their quarterly newsletter, *The Minor Planet Bulletin*. Should our article be accepted for publication, it would most likely be published in

August. Co-authoring the article was Maj. Charles Wetterer. A copy of our submitted paper can be found in Appendix B.

CHAPTER V

1035 AMATA ASTROMETRY OBSERVATION PROCEDURES

Selecting 1035 Amata

CCD Astrometry was conducted to determine the right ascension and declination of several asteroids. The first part of this process was to select an asteroid or asteroids on which to conduct astrometry. It was decided to use the critical list of asteroids published by the Minor Planet Center to determine which asteroids to observe. The critical list is a regularly updated publication, which is used by astronomers to select asteroids for observing. The basic reason that asteroids are put on the critical list is because there are relatively few observations of that asteroid, and more are needed to get an accurate estimation of the orbit. Specifically, there are six different categories of asteroids on the critical list.[10] The categories are broken down based on the number of oppositions the asteroid has been observed at over a certain time frame. The specific asteroid that we eventually selected was 1035 Amata, although it wasn't selected until later. Amata was listed in Category 5 of the critical list which is for objects observed at four or more oppositions only one night in the last ten years.[10] Opposition refers to the asteroid's location in its orbit with respect to the Earth and the Sun. Since the asteroid's orbit lies outside the Earth's, opposition occurs when the Earth and asteroid both lie on the same side of the Sun. Conjunction, the opposite of opposition, occurs when the Sun lies between the Earth and the asteroid. See Fig 5.1 for an illustration of this concept.

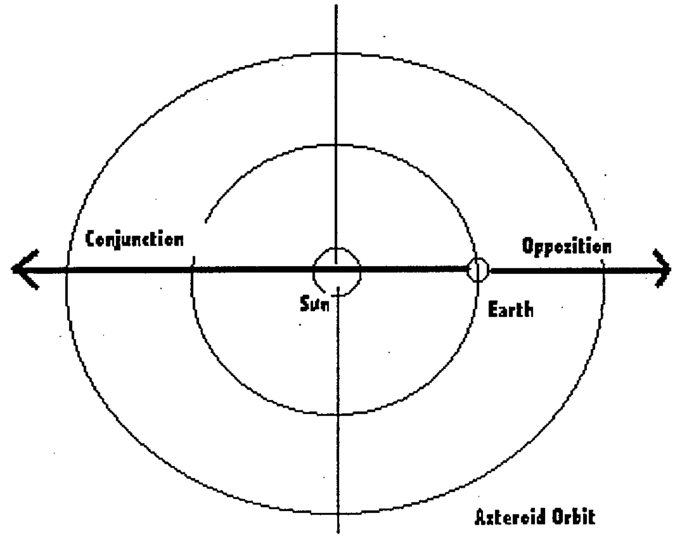

Fig. 5.1 Opposition and Conjunction Positions

There were two reasons that we decided to view asteroids from the critical list. First, it could be assumed that the critical list asteroids would be in favorable viewing positions during our observation times. In addition to obtaining data useful for this thesis, the data we collected could be reported to the Minor Planet Center for use in updating their asteroid ephemerides. Starting with the critical list we next consulted Guide to see when specific asteroids would be visible during our viewing time.[2] Guide allows the observer to select the asteroid number, which it then shows in relation to other nearby stars at the time specified. It was possible to set the time to when we would be observing to see if there were favorable comparison stars in close vicinity to the asteroid. At least four comparison stars are highly recommended and seven to twelve comparison stars are preferred by the analysis software used to determine the position of the asteroid. [13]

Special consideration was given to the fact that the asteroids would have some relative motion over the period that we would be observing it. It was therefore necessary to make sure that the asteroid would not be passing too close to other stars over the observing time. In order to perform the astrometry we would need two pairs of observations separated by approximately one hour. The reason for this separation is so that visual inspection could ensure that the object suspected to be the asteroid was in fact our target asteroid. Since the asteroid will have relative motion over one hour and stars do not, the object that has moved from one observing time to the next is the target asteroid.

Another area of consideration was the magnitude of the asteroid. Most asteroids, because of their distance from the Sun and size, will have magnitudes in the range of 13 to 22. Due to the limitations of our telescope, we decided a reasonable upper magnitude limit would be about 19 magnitudes. The problem with observing dimmer asteroids is that it might be difficult to get an accurate image with an exposure time of only a few minutes.

Initially several asteroids were selected for observing. This was done to provide backup data should the data on any specific asteroid not be as good as another was. This allowed us to wait until later to select the specific asteroid that we would use for orbit determination. Additionally any observations we could make would be useful to the Minor Planet Center. Since there needed to be an hour separation between pairs of observations for each asteroid, we could make better use of that time by imaging other asteroids.

Observing Asteroids

The observations were taken using a 41-cm Cassegrain telescope and an ST-8 CCD camera. This CCD was electronically cooled to –10°C. The temperature was specified with Sky-Pro CCD Astronomy Software which is also the software that took the exposures.[11] After the telescope was initialized to know what coordinates it was pointing at, it was necessary to focus the telescope. The Sky-Pro software has a focusing option to assist in obtaining a good focus. A random star is chosen and highlighted on the computer screen. Short, repeated exposures are taken automatically by the camera while the operator adjusts the focus. By visual inspection of the images resulting from the different focuses, the operator can select the best one.

To initially locate the asteroid the telescope was slewed to the right ascension and declination given by Guide. An exposure just long enough to pick up the stars in the field of view (approximately 20 seconds) was taken. This image was then compared against the picture given by Guide. Some offset corrections were necessary to center the asteroid and the planned comparison stars in the field of view.

For most of the asteroids observed a two-minute exposure time was sufficient. For one asteroid with a higher magnitude, a three-minute exposure was taken. For each observation taken there were two frames that were exposed, a light frame and a dark frame. A light frame is obtained by actually opening the camera shutter and collecting photons from the sky. The dark frame is an exposure of the same length as the light frame, but is taken with the shutter closed. The dark frame is necessary since the camera is not operating below –100°C.[6] It is needed to determine the amount of dark current or thermal noise in the system and the DC offset voltage (bias) applied to the CCD, which is later subtracted out. This exposure takes into account the instrument bias, so a separate bias frame is not needed. The Sky-Pro software automatically takes a dark frame of equal length for each light frame taken. In order to save time during observations, one dark frame of the appropriate length was taken which could later be subtracted from all of the light frames. Each exposure then needed only a light frame.

The other type of exposure needed was a flat field image. These images are typically taken over a one to two second exposure time. The exact duration is not as important as all flat field exposures being of the same length. Flat field images are exposures taken of an illuminated background to determine the pixel-to-pixel sensitivity, or the quantum efficiency of the CCD chip.[6] Depending on when the observations were made on a particular evening, the flat field images could be taken of either the illuminated inside surface of the dome, or the twilight sky. Over the course of the observations, flat fields were taken using both techniques. All of the flat field images (typically three to five) were averaged to create a master flat field image. This flat field correction is then applied to the light frames after the dark frames have been subtracted.

Two observations of each asteroid observed that evening were taken. Although only one observation was technically needed, two were taken mainly for redundancy and backup purposes. After approximately one hour, we returned to the first asteroid viewed so that we could repeat the observation cycle. Since some asteroids rose and set before others, we planned our observations so that those that rose (and therefore set) earliest were imaged first and then observed the other asteroids in a logical sequence.

The observatory log for the astrometry was filled out with the same information as recorded with photometry. Reference Chapter II for a more thorough discussion of that data. The only difference was that no filter was used for any of the observations.

To go back for the second set of observations of an asteroid, we needed to use the coordinates that the telescope was actually pointing at when the image was taken rather than the Guide Star Catalog coordinates. This is because the telescope position was usually offset slightly from the Guide Star Catalog coordinates and we wanted to return to the exact coordinates used previously. Furthermore the Guide Star Catalog gives coordinates in the J2000 epoch and the telescope gave the epoch as the present time, J1998 in this case.

Verifying Capture of Target Asteroid

After the new images were taken, one of the earlier images was opened for a visual comparison. Although the asteroid had moved over this time interval, sometimes it was difficult to notice the change since it didn't move particularly far. Noticing the change was especially difficult in an image 'cluttered' with other stars and objects. Two example images of 1035 Amata (indicated by the arrow) are shown in Fig. 5.2 and Fig. 5.3, which were taken at different times. The Sky-Pro software allowed us to do a blink comparison of the two frames. A star common to both frames was selected and its pixel coordinates (not to be confused with celestial coordinates) in each frame were noted. An offset was then applied to one picture to get both pictures to align. By then rapidly blinking the images back and forth, we could visually see which object changed positions, thus identifying it as our asteroid. The target asteroid was usually quite easy to identify. However, in the interest of astronomy, each frame was carefully inspected for other objects that had changed positions. The purpose of this was to see if we had by chance happened to catch another previously unknown asteroid. None of our frames indicated such a discovery.

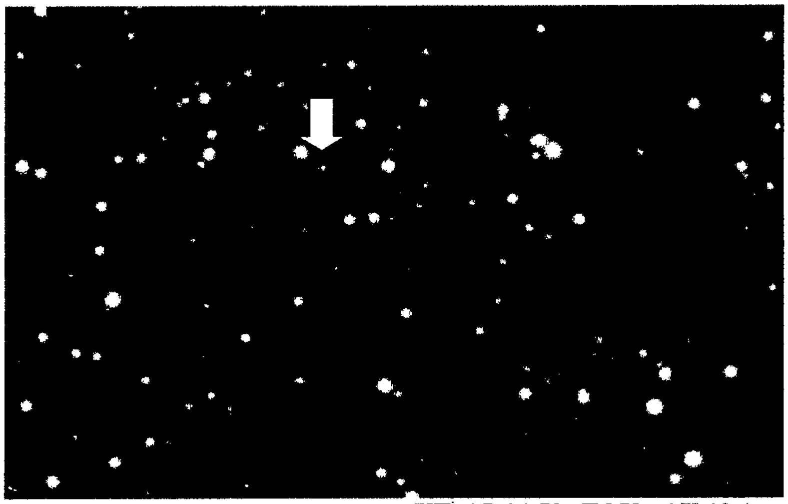

Fig. 5.2 ST8 Image of 1035 Amata. UT: 05:46:58. FOV: 6 X 10 Arcminutes

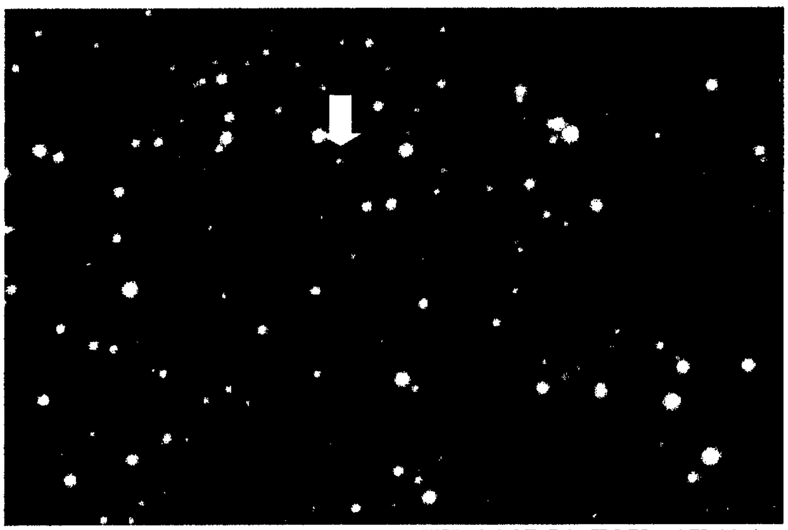

Fig. 5.3 ST-8 Image of 1035 Amata. UT: 06:27:56. FOV: 6 X 10 Arcminutes

After the appropriate data reduction we decided to use 1035 Amata as the asteroid for the orbit determination. Orbit determination would require observations over several weeks and Amata was favorably positioned for such observations. In addition, it had a relatively low magnitude of about 16. Furthermore, the published orbital elements for Amata were found on the Internet, which would allow us to verify our calculated elements with the approved elements. In total, six nights of observations were collected on 1035 Amata. In addition to the author, data was also collected by the Physics 480 class at the Air Force Academy. A summary of the six observation nights is presented in Table 5.1. Note the start time refers to the first image of Amata and the stop time refers to the final observation of Amata, not necessarily of the entire evening. The right ascension and declination refer to the telescope pointing coordinates, not Amata specifically.

Date	Start time	Stop time	# of Obs	RA J1998	Dec J1998
UT 98 Jan 21	UT 5:48	UT 6:35	4	3 44 42.4	42 12 32
UT 98 Jan 22	UT 5:12	UT 6:44	4	3 45 12.5	42 08 35
UT 98 Jan 27	UT 5:10	UT 6:46	6	3 45 32.8	41 42 54
UT 98 Jan 28	UT 5:07	UT 6:30	6	3 45 34.4	41 38 00
UT 98 Feb 13	UT 1:37	UT 3:26	6	3 53 13.2	40 30 05
UT 98 Mar 13	UT 2:11	UT 3:23	6	4 18 21.0	39 20 08

Table 5.1. Astrometry Observing Log. RA given in hour, min, sec. Dec given in °, ','"

CHAPTER VI

1035 AMATA DATA REDUCTION

Image Corrections

After all of the observations have been collected, initial processing is performed. This involves the dark frame and master flat field image discussed in the previous chapter. These procedures are performed with the Sky-Pro software. The dark frame is simply subtracted from each light frame. What is actually being subtracted is the exact pixel value of each pixel in the dark frame from the corresponding pixels in the light frame. Flat field corrections are applied in a more complex manner. For visualization purposes, the flat field can be thought of as being divided into the light frames. An example of a flat field image can be seen if Fig. 6.1. It is usually good practice to save both the raw images and the corrected images should the processing need to be reaccomplished for whatever reason.

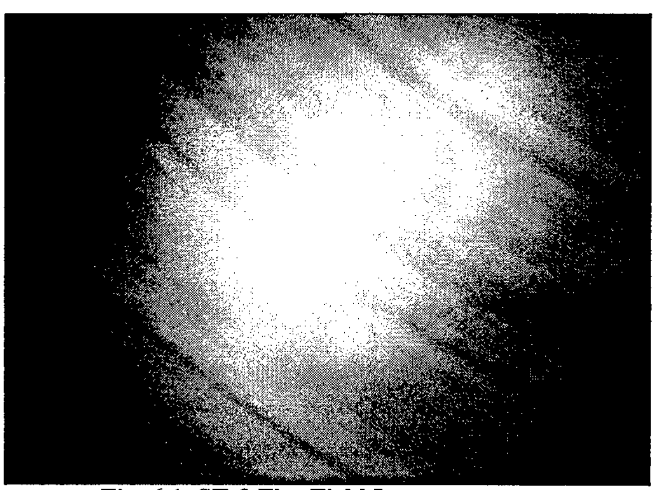

Fig. 6.1 ST-8 Flat Field Image

Astrometry Software

After this initial processing the images are ready to be processed to determine the right ascension and declination. The software used to do this is called Astrometrica, version 3.1b.[12] Astrometrica is a shareware program but the author requests purchase for extended use and full capabilities.[13] The remainder of this chapter will summarize the operation of Astrometrica, its capabilities, credentials, and the mathematical techniques it employs. It should be noted that for our purposes Astrometrica was used to perform astrometry of asteroids, but it can also be used for comets, moons, and variable stars to name a few of its other uses. For more information, visit either the Astrometrica home page [12] or the Astrometrica user manual. [13]

Astrometrica was originally created using Borland Pascal 7.0©. It was created to be a useful tool for both amateur and professional astronomers to perform CCD astrometry. It is currently used in 30 countries on seven continents and has been used for such purposes as discovering a new satellite of Uranus and the first Allen-type asteroid discovered by an amateur astronomer. The Astrometrica home page identifies the Winter 1995 issue of "CCD Astronomy" magazine, pp. 20-22, as being a source of further information.[12]

When first using Astrometrica the location of the observatory must be programmed. This data is used to calculate the coordinates of the asteroid and is included in the report generated by Astrometrica. The corrected ST-8 images are first opened into Astrometrica. Astrometrica is capable of reading several different data formats, including Santa Barbara Instrument Group (SBIG), the format used by the ST-8. If necessary, a smoothing or median filter function can be used to correct any galactic or cosmic ray pixel spikes. This function is useful for calculating magnitudes, but since that was not the primary objective of our observations, it was not performed on any of our images.

Performing Astrometry

The first step in the data reduction is to identify the background brightness. A box appears on the screen, which is moved over an area with no stars as close to the target object as possible. Next, the position and brightness of the target object is identified using two similar boxes. The box size should be adjusted to encase the entire object, yet being as close a fit as possible. The purpose for identifying the background brightness is similar to that for obtaining a dark frame exposure. The average brightness value inside the box is subtracted from the target object and reference stars.

After identifying the target object, the reference or comparison stars need to be identified. Astrometrica uses the Hubble Guide Star Catalog as its source of reference stars which is convenient for our purposes since we used that software in locating the asteroid for observations. A library image is opened on the screen, which identifies the stars for which Astrometrica (via the Guide Star Catalog) knows the locations. It is necessary to determine which stars in the library image correspond to the stars in our CCD images. A box similar to the one used earlier is used to select the reference stars. At least four are highly recommended and seven to twelve are preferable, provided there are that many in all of the CCD images and that they do not have close companion stars. It is necessary to identify the reference stars in both the library image and the CCD image, being careful to select them in the same order each time.

A centroid for the object and each reference star is calculated based on a center of gravity calculation using the number of photons collected on the CCD pixels. A contrast index is then calculated from a linear regression. This index is used to create a brightness ratio for each reference star, which is compared against the Guide catalog information. The data from each reference star is compiled into one single, virtual reference star, which is used to calculate the magnitude of the target object. The coordinates are calculated by means of a least squares fit. This produces a set of plate constants that can be translated into rectangular coordinates. A series of coordinate transformations then adjusts these coordinates to right ascension and declination.

The only other user inputs required are some of the observing log parameters such as asteroid name, exposure duration, and the universal time at the midpoint of the exposure. Astrometrica calculates the right ascension, declination, and magnitude from this information. The precision of the right ascension given is 0.001 s and the precision of the declination is 0.01", although the final digit is not significant for absolute positions. The results are outputted using a report file in a format preferred by the Minor Planet Center.

Reporting Observations to the Minor Planet Center

By adding appropriate header information, this report can be e-mailed to the Minor Planet Center, which then adds the observations to their database to update the ephemeris on that asteroid. For more information on the report format, reference the Minor Planet Center Internet site.[14] Each observatory is given an identification number that is published in the Minor Planet Center circulars and Internet web pages to acknowledge the contributions from that observatory. The code for the U.S. Air Force Academy Observatory is 712 and can be seen on the Minor Planet Center Internet site.[10] All of our observations were submitted in two separate data batches to the Minor Planet Center. An example of one of these submissions can be found in Appendix K. It is not in the specific format requested by the Minor Planet Center, but it is summarized to make it easier to understand.

The validity of all observations sent to the Minor Planet Center is confirmed by comparing them to the existing data for that asteroid. Gareth Williams, the Associate Director of the Minor Planet Center, explained in an e-mail that all of the observations are propagated at a nearby 200-day epoch.[15] The resulting orbit is checked to see if it falls within the tolerances of their existing orbit data. The exact tolerance was not specified. A return e-mail was sent to us indicating which of our observations had unacceptable error. Of all the observations submitted by the USAFA Observatory, only one was returned as invalid and that was for a different asteroid than Amata. The reason for the error in that observation could possibly be do to slight errors in the Guide Star Catalog coordinates for the comparison stars, but the exact reason is unknown. The confirmation of the validity of our data is significant. It verifies that the observational and data reduction procedures followed were accurate and that the object we suspected was 1035 Amata was in fact that object. Verifying the validity of the data allowed us to proceed in determining the orbital elements.

CHAPTER VII

1035 AMATA ORBIT DETERMINATION

Gauss' Method of Angles Only Orbit Determination

Thirty-two observations of 1035 Amata were collected from six nights of observations. These data are presented in Appendix C. The data included are the year; month; day and decimal part; right ascension in hours, minutes, seconds; and declination in degrees, minutes, seconds. The computed absolute magnitudes and the filter used for the observations are also included. Since none of the astrometry observations used a filter, the V represents visible. The magnitudes and filter were deleted, however, before the file was used by the orbit determination software. The numbers preceding certain lines indicate which input files those data were used in. For example, the three lines with a preceding '1' were all used for the first input test case in the order of earliest to latest observation date. These numbers are for reference only and were not used in the actual input file.

Advising in the orbit determination for 1035 Amata was Mr. Roger Mansfield. He recommended that Gauss' method of angles only orbit determination would be appropriate for this problem since the data included three position measurements (angles) and the times of the observations. This method had an equation with six unknowns, which must be solved to determine the orbit. Since each observation provides two angles, a minimum of three observations are required. Unless otherwise noted, the following summary of Gauss' angles only method of orbit determination is taken from J.M.A. Danby's *Fundamentals of Celestial Mechanics*.[16]

The first consideration in using this approach is that for some problems a solution cannot be found. This can result from inaccurate position measurements or an unfavorable geometry in the relative positions of the observations to each other. Additionally, for some deep space objects with peculiar orbits, an orbit may simply not be determinable with this method. Gauss assumes that the three observations lie in a plane. This assumption would not be valid if third-bodies were exerting perturbing forces on the asteroid, which would change its orbit plane. Although several thousand asteroids have orbits close to Amata (celestially speaking), the small mass of these objects is not likely to exert a large perturbing force. Perturbations from larger objects such as Jupiter or Mars are probably not a concern over the short time interval (three months) during which the observations were made. The relative positions of Earth, Jupiter, Mars and Amata at the time of observation are unknown.

Since all the vectors lie in the same plane, it is possible to define one of them, the middle observation, in terms of the other vectors with two scalar constants.

$$\mathbf{r}_2 = \mathbf{c}_1\mathbf{r}_1 + \mathbf{c}_3\mathbf{r}_3 \tag{7.1}$$

One of the most difficult aspects of Gauss' method is selecting the initial values for these scalars. Danby recommends using the opening terms from the **f** and **g** series to select these values.

$$c_1 = g_3 / (f_1 g_3 - g_1 f_3) \tag{7.2}$$

$$c_3 = - g_1 / (f_1 g_3 - g_1 f_3) \tag{7.3}$$

After successive iterations, the constants are updated, but an accurate initial guess is crucial in determining whether or not the process will converge.

These constants are then used in calculating Gauss' sector-triangle ratios. This is a ratio of the entire area swept out by the asteroid orbit between observations to the area of the triangle swept out between observations. The concept behind this calculation is Kepler's Second Law, which equates equal areas in equal times.[17] An illustration of this ratio is provided in Fig 7.1. Sector-triangle ratios are computed for observations 1 to 2, 2 to 3, and 1 to 3. Fig.7.1 shows the calculation for observations 1 to 3. A series of coordinate transformations are then performed to convert the observations from the equatorial system to a new system. A similar transformation is performed on the Sun coordinates to be in the same reference system as the observations. After this, the geocentric distances to the asteroid are calculated as follows

$$\rho_2 = (-c_1 Z_1 + Z_2 - c_3 Z_3) / \upsilon_2 \tag{7.4}$$

$$\rho_3 = (\rho_2 \mu_2 + c_1 Y_1 - Y_2 + c_3 Y_3) / c_3 \mu_3 \tag{7.5}$$

$$\rho_1 = (\rho_2 \lambda_2 - c_3 \rho_3 \lambda_3 + c_1 X_1 - X_2 + c_3 X_3) / c_1 \tag{7.6}$$

where λ, μ, and υ are components of the unit vectors in the direction of the observations and **X**, **Y**, and **Z** are the solar coordinates.

The process is repeated until the change in the new geocentric distances is less than a specified tolerance. From the final range distance, the desired orbital elements are calculated.

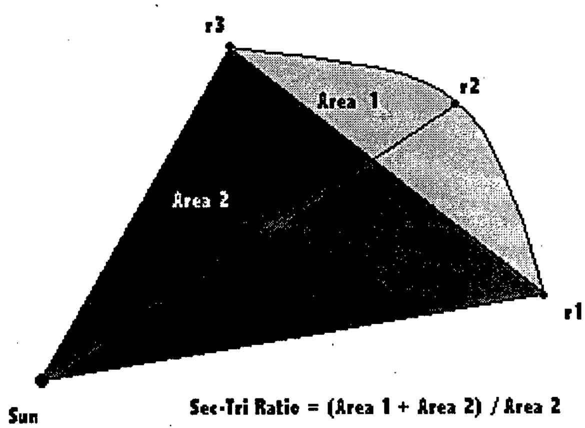

Fig. 7.1 Example Sector Triangle Ratio

GEM Software

It was decided to use the set of computer programs for calculating orbital elements that came with Danby's *Fundamentals of Celestial Mechanics*.[16] The specific program of interest was the Gauss-Encke-Merton (GEM) method, named after its three main contributors. The Turbo Pascal version of this program was used. To begin, the unit vectors for the direction cosines of the observations, and the Sun coordinates at the observation times are calculated. The exact Sun coordinates are calculated based on the Julian Date of the observation and tabular information provided by Bretagnon and Simon. These are both calculated in the ecliptic plane with an obliquity of the ecliptic unique for each observation Julian Date.

By Danby's admission, the GEM program is not a completely accurate program since it does not account for planetary aberration. Danby also indicates that more accurate Sun position angles should be determined.[18] Planetary aberration indicates that the time an observation is made does not indicate the true position of an object for that recorded time since the light reflected from it travels to the Earth with a finite speed. Despite its potential shortcomings, the GEM program was attempted and appropriate modifications and corrections were made where needed.

Some of the initial modifications made were to add two additional data input procedures, which allowed the data to be entered either from the keyboard as the program was running or to read data from an external input file. Additionally a declination conversion was added to allow the declination to be input as degrees, minutes, seconds and then converted to degrees and fractional degrees. This is the format provided by the astrometry process and most commonly used by astronomers. A user query was also added after each iteration loop to ask the user if it should continue with the next iteration. The data printed to the screen at each query allowed to user to see if the program was converging or not. The specific values checked for convergence were the geocentric distances and the sector-triangle area ratios. There were three separate values for each of these, one for each input observation. All external procedures were also incorporated into the main GEM program file so separate procedure files were not needed. Comments and structure changes were also made throughout to make the program easier to read and understand. The GEM program with all of our modifications can be found in Appendix D.

Since the data included thirty-two observations of Amata, six sets of random combinations of three observations each were created. In order to ensure sufficient time and distance between observations, multiple observations from the same evening were not used. In general, an observation was selected from the beginning, middle, and end of the data list. The actual data sets used are numbered in Appendix C.

The program did converge for these data and was close, but not identical to the orbital elements published by the Minor Planet Center. The published orbital elements from the Minor Planet Center can be found in Table 7.1. Mr. Gareth Williams explained that the Minor Planet Center uses several different orbit determination algorithms, but most of them are a derivative of Gauss' method.[15] The calculated elements from six test cases using the GEM program can be found in Table 7.2. The complete output files from the GEM program can be found in Appendix E. To find errors in the program, hand calculations were performed using the Amata data. An error in the calculation of one of the arrays was found in the program. Correcting it did not significantly improve the results however, since the error was in the direction of the **K** unit vector and the inclination in this direction is less than 20°. A modified subroutine to calculate the Sun angles that converted all the elements to the J2000 epoch was also tried but it failed to significantly improve the results. No attempt to account for planetary aberration was made since it was felt that this too would provide only minimal improvements in the accuracy.

Minor Planet Center			
a (AU)	3.137178	Incl. (°)	18.08732
P (years)	5.56	M (°)	85.82541
E	0.2026701	Ω (°)	2.20159
n (°/day)	0.17737536	ω (°)	323.12242

Table 7.1 Minor Planet Center Orbital Elements [19]

GEM	Input 1	Input 2	Input 3	Input 4	Input 5	Input 6
a (AU)	3.2182661	3.2101537	3.2268884	3.2727042	3.2071182	3.2503599
P (years)	5.7735259	5.7517096	5.7967439	5.9206355	5.7435533	5.8601047
e	0.1597266	0.1632919	0.1559225	0.1237473	0.1660712	0.1431401
n (°/day)	0.1707148	0.1713623	0.1700310	0.1664731	0.1716057	0.1681926
Incl. (°)	18.062012	18.063142	18.061130	18.049346	18.061310	18.053590
M (°)	-----	-----	-----	-----	-----	-----
Ω (°)	1.4028082	1.4833787	1.3648144	0.9540862	1.5977124	1.1920249
ω (°)	329.27072	328.62022	329.91056	334.45223	328.51103	332.18644

Table 7.2 GEM Orbital Elements. J2000 Epoch

ORBDET Software

It was then decided to try the ORBDET orbit determination program by Montenbruck and Pfleger.[17] Without any modifications, this program yielded results that were significantly closer to the published values than the GEM program did. The orbital elements calculated by ORBDET from the six test cases can be found in Table 7.3. A slight modification to the input data needed to be made for the ORBDET program. The decimal part of the day was multiplied by 24 to give hours and decimal hours.. An example of an input file for ORBDET can be found in Appendix G. The complete output files from ORBDET can be found in Appendix H. ORBDET also accounted for planetary aberration.

ORBDET	Input 1	Input 2	Input 3	Input 4	Input 5	Input 6
a (AU)	3.139193	3.139943	-0.733254	3.140728	3.140653	3.141972
P (years)	5.5621	5.5640	0.6279	5.5661	5.5659	5.5694
E	0.202770	0.202402	4.394909	0.202682	0.202028	0.201908
n (°/day)	0.177205	0.177142	1.569720	0.177076	0.177082	0.176970
Incl. (°)	18.0870	18.0866	18.6437	18.0868	18.0864	18.0861
M (°)	85.670976	85.671276	-332.9817	85.575542	85.670941	85.595332
Ω (°)	2.1808	2.1676	340.4056	2.1664	2.1577	2.1416
ω (°)	323.2704	323.3218	158.9076	323.3803	323.3711	232.4682

Table 7.3 ORBDET Orbital Elements. J2000 Epoch

With this information, we decided to abandon further attempts at debugging the GEM program. In general, the mathematical techniques employed in Danby's text are thought to be more robust than those used by Montenbruck and Pfleger. Montenbruck and Pfleger however, are more concerned with reliable software development. Therefore even though mathematically not as robust as GEM, ORBDET has been properly debugged, yielding results that are more accurate.

The following description of the ORBDET program comes from the Montenbruck and Pfleger text.[17] ORBDET uses Bucerius' method, which is derived from Gauss' method of angles only orbit determination but with more simplifications. The heart of Gauss' method, the sector-triangle ratio calculations, is used here as well as in GEM. The main simplification in Bucerius' method to Gauss' method is that it is unable to account for orbital paths crossing a tear-drop shaped region of the space around the Sun and inside the Earth's orbit defined by Charlier's boundary line. Fig. 7.2 illustrates this region. Observations from this region could have two possible solutions which Bucerius' method cannot handle. An object with an orbit crossing this region might be a near earth asteroid or comet with a high eccentricity. However, since the orbit of Amata never takes it inside this region, it can always be observed at opposition, which the Bucerius' method is capable of handling. Although ORBDET cannot handle these situations, it does flag when such a situation occurs. This would allow the user to attempt a more complete method of orbit determination.

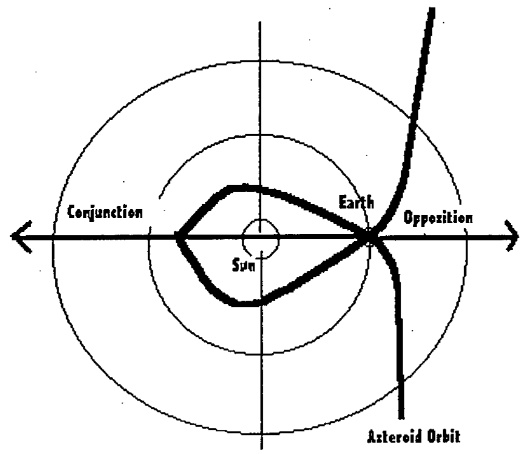

Fig. 7.2 Charlier's Boundary [17]

Additional calculations for other orbital elements were added to ORBDET to allow us to compare with the Minor Planet Center elements. Those equations for the mean motion, mean anomaly, and the period are

$$\mathbf{n = kg * a^{(-3/2)}} \tag{7.7}$$

$$\mathbf{M = n * (JD_2 - JD_{MPC})} \tag{7.8}$$

$$\mathbf{P = (360 / n) * (1 / 365.25)} \tag{7.9}$$

where **kg** is the Gaussian constant, $\mathbf{JD_2}$ is the JD for the second observation, and $\mathbf{JD_{MPC}}$ is the Minor Planet Center epoch

Notice that the mean motion was corrected to the epoch used by the Minor Planet Center to allow for a direct comparison of this parameter. The main program that calculates the orbital elements can be found in Appendix F. ORBDET uses a large number of functions and procedures found in external files. To see these files reference the software associated with Montenbruck and Pfleger's text. [17]

FIND_ORB Software

A last minute discovery was made in the preparation of this thesis. A batch least squares program was found on the Internet, which was capable of handling an entire list of observations. The name of this program is FIND_ORB, written by Bill Gray and maintained at the Project Pluto web page.[20] FIND_ORB is ideal software for astronomers who submit data to the Minor Planet Center. The output format is identical to that used in the Minor Planet Center circulars. The epoch used by the Minor Planet Center can also be directly input to allow direct comparison of all the computed elements.

FIND_ORB initially assumes a distance of 1 AU from the Sun to the first and last observations. Through iterations, this distance is modified. The program will converge faster if a more accurate initial distance guess is input. This method is referred to as the Herget method, which is used to get an initial estimate of the orbit. A residual is calculated for each iteration, which is the observation values minus the calculated values. The Herget step is repeated until the residuals aren't decreasing anymore. With this estimation of the orbit a full step is taken which involves the least squares "best fit" process. A few full steps are performed until the residuals fail to decrease any further.

FIND_ORB also has the capability of allowing for perturbations of all the planets and the moon, although each body selected increases the computing time required for each iteration. It is interesting that including all of the possible perturbations did not decrease the residuals any further. This would indicate that the two-body assumption for Gauss' method was valid. A possible reason third-body affects weren't noticed is that the observations were collected over only a couple of months. Had observations been made over several years, perturbation effects probably would have been noticed.

Bill Gray, the author of FIND_ORB indicates that residuals of less than one arcsecond would indicate quite accurate observations.[20] For our observations, FIND_ORB calculated a final residual of 0.360 arcseconds, which is another confirmation of the validity of our observational and data reduction procedures. The calculated elements matched the MPC values to about four or five decimal places for most parameters.

The average values of the computed elements from each method used are shown in Table 7.4. Note that the ORBDET elements from test case 3 were not used because ORBDET failed to find a reasonable solution for the orbit for that test case. The three programs used are also arranged in order of increasing accuracy.

Averages	GEM	ORBDET	FIND_ORB	MPC
a (AU)	3.2317204	3.1404978	3.1373621	3.137178
P (years)	5.8099058	5.5655000	5.56	5.56
e	0.1511954	0.2023580	0.2025591	0.2026701
n (°/day)	0.1696697	0.1770950	0.17736059	0.17737536
Incl. (°)	18.0578800	18.0865800	18.08732	18.08732
M (°)	-------	85.6368134	85.82430	85.82541
Ω (°)	1.3260021	2.1628200	2.20044	2.20159
ω (°)	330.608128	305.162360	323.13457	323.12242

Table 7.4 Average Orbital Elements

CHAPTER VIII

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

Photometry Results

Differential photometry proved to be an effective method for calculating the rotational period of an asteroid. To restate the results of our work, a 9.210 ± 0.005-hour synodic period was calculated for 583 Klotilde. We have a high level of confidence in our method and procedures at arriving at this period. Of course with any observational procedure, there is the possibility of error especially considering that visual judgments were made for some of our conclusions. We do not feel however, that these judgments have invalidated our conclusions.

Additional Photometry Research

There are still numerous asteroids in our solar system in a variety of orbits both close to and distant from the Earth that have not had their rotational periods calculated. Additional asteroids have had preliminary work done, but no final rotational period has been determination. The procedures outlined in this thesis could be duplicated for many of those asteroids.

Of particular interest in our research was the indication of discovering a previously unknown variable star. Further observations and analysis are needed to verify our initial data and characterize the nature of this star's variability. The star in question is GSC 223:1761 (Guide Star Catalog designation) located in the constellation Cancer.

Astrometry Results

The observational techniques and data reduction procedures used to determine asteroid positions were verified by confirmation from the Minor Planet Center as well as the ORBDET and FIND_ORB orbit determination programs. The results obtained through ORBDET closely matched the elements calculated by the Minor Planet Center and the batch least squares method used by FIND_ORB yielded even closer results. The GEM program also has potential to be an accurate method of orbit determination, but further debugging needs to be done on it first. Although Gauss' method of angles only orbit determination worked for our data there may be orbits where this method will not converge and it will not be possible to determine an object's orbit.

Additional Astrometry and Orbit Determination Research

There are also several areas for further research in astrometry. The Minor Planet Center continuously updates its critical list of asteroids. They are always interested in observations from amateur astronomers to augment their ephemerides. Coordinates can be computed for any of these asteroids and reported to the Minor Planet Center without having to perform the orbit determination done in the project. Similar work can also be accomplished for comets. There are undoubtedly hundreds of other minor planets and comets in our solar system that have not yet been discovered. An observant astronomer might be fortunate enough to discover a new comet or asteroid.

The orbit determination methods used here also suggest areas of additional work. ORBDET was only modified to calculate additional orbital elements for comparison with the Minor Planet Center elements. Since our database of observations was quite large, the program could be modified to read in several observations and apply a batch least squares approach to average them. A similar process could be accomplished with GEM once all the bugs in the program have been removed.

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APPENDIX A

Differential Photometry Spreadsheets

583 Klotilde photometry (UT 98 Feb 13)

														offset																
	|<--------				instrumental magnitudes				----------->|					14.5				|<-- differential photometry (adjusted) ----->											-0.937	-0.364
image	Klotilde		1		2		3						Time	comb		image		Klot-1		Klot-2		Klot-3		1-2		1-3		Time	1-2	1-3
	mag	err	mag	err	mag	err	mag	err	h	m	s	exp	(hours)	mag	err			mag	err	mag	err	mag	err	mag	err	mag	err	(hours)	mag	mag
17	12.474	0.003	13.776	0.008			14.031	0.010	4	43	17	300	4.763	13.198	0.009	17	13.776	-1.302	0.009			-1.193	0.010			-0.255	0.0128	4.763		0.109
18	12.492	0.003	13.804	0.009			14.055	0.010	4	49	7	300	4.860	13.188	0.009	18	13.804	-1.312	0.009			-1.199	0.010			-0.251	0.0135	4.860		0.113
19	12.461	0.003	13.761	0.008			13.999	0.009	5	1	14	300	5.062	13.200	0.009	19	13.761	-1.300	0.009			-1.174	0.009			-0.238	0.012	5.062		0.126
20	12.476	0.003	13.784	0.008			14.018	0.009	5	8	10	300	5.178	13.192	0.009	20	13.784	-1.308	0.009			-1.178	0.009			-0.234	0.012	5.178		0.130
21	12.494	0.003	13.798	0.008			14.016	0.009	5	19	28	300	5.366	13.196	0.009	21	13.798	-1.304	0.009			-1.158	0.009			-0.218	0.012	5.366		0.146
22	12.506	0.003	13.805	0.008			14.019	0.009	5	25	37	300	5.469	13.201	0.009	22	13.805	-1.299	0.009			-1.149	0.009			-0.214	0.012	5.469		0.150
23	12.533	0.003	13.815	0.008			14.046	0.009	5	32	22	300	5.581	13.218	0.009	23	13.815	-1.282	0.009			-1.149	0.009			-0.231	0.012	5.581		0.133
24	12.565	0.003	13.832	0.008			14.055	0.009	5	44	1	300	5.775	13.233	0.009	24	13.832	-1.267	0.009			-1.126	0.009			-0.223	0.012	5.775		0.141
25	12.542	0.003	13.761	0.007			14.681	0.015	5	52	5	300	5.910	13.281	0.008	25	13.761	-1.219	0.008			-1.775	0.015			-0.920	0.0166	5.910		-0.556
26	12.673	0.003	13.894	0.007			14.831	0.017	5	58	44	300	6.021	13.279	0.008	26	13.894	-1.221	0.008			-1.794	0.017			-0.937	0.0184	6.021		-0.573
27	12.814	0.003	14.011	0.007			14.293	0.009	6	6	10	300	6.144	13.303	0.008	27	14.011	-1.197	0.008			-1.115	0.009			-0.282	0.0114	6.144		0.082
28	12.670	0.004	13.872	0.010	14.767	0.021			6	22	0	300	6.408	13.298	0.011	28	13.872	-1.202	0.011	-1.160	0.021			-0.895	0.023			6.408	0.042	
29	12.843	0.005	14.009	0.012	14.959	0.028			6	28	11	300	6.511	13.334	0.013	29	14.009	-1.166	0.013	-1.179	0.028			-0.950	0.030			6.511	-0.013	
30	12.640	0.003	13.846	0.009	14.708	0.018			6	34	7	300	6.610	13.294	0.009	30	13.846	-1.206	0.009	-1.131	0.018			-0.862	0.020			6.610	0.075	
31	12.742	0.004	13.947	0.010	14.805	0.022			6	41	0	300	6.725	13.295	0.011	31	13.947	-1.205	0.011	-1.126	0.022			-0.858	0.024			6.725	0.079	
32	12.753	0.004	13.952	0.010	14.802	0.022			6	47	0	300	6.825	13.301	0.011	32	13.952	-1.199	0.011	-1.112	0.022			-0.850	0.024			6.825	0.087	
33	12.677	0.004	13.900	0.009	14.797	0.018			6	53	51	300	6.939	13.277	0.010	33	13.900	-1.223	0.010	-1.183	0.018			-0.897	0.020			6.939	0.040	
34	12.605	0.003	13.821	0.008	14.774	0.017			6	59	53	300	7.040	13.284	0.009	34	13.821	-1.216	0.009	-1.232	0.017			-0.953	0.019			7.040	-0.016	
35	12.565	0.003	13.777	0.008	14.703	0.017			7	5	43	300	7.137	13.288	0.009	35	13.777	-1.212	0.009	-1.201	0.017			-0.926	0.019			7.137	0.011	
36	12.610	0.003	13.834	0.008	14.746	0.018			7	11	30	300	7.233	13.276	0.009	36	13.834	-1.224	0.009	-1.199	0.018			-0.912	0.020			7.233	0.025	
37	12.598	0.003	13.835	0.007	14.755	0.016			7	18	10	300	7.344	13.263	0.008	37	13.835	-1.237	0.008	-1.220	0.016			-0.920	0.017			7.344	0.017	
38	12.745	0.003	13.988	0.007	14.901	0.014			7	23	55	300	7.440	13.257	0.008	38	13.988	-1.243	0.008	-1.219	0.014			-0.913	0.016			7.440	0.024	
39	12.894	0.003	14.135	0.007	15.050	0.015			7	29	30	300	7.533	13.259	0.008	39	14.135	-1.241	0.008	-1.219	0.015			-0.915	0.017			7.533	0.022	
40	12.628	0.003	13.873	0.008					7	35	10	300	7.628	13.255	0.009	40	13.873	-1.245	0.009									7.628		
41	12.591	0.003	13.851	0.008					7	40	45	300	7.721	13.240	0.009	41	13.851	-1.260	0.009									7.721		
42	12.620	0.003	13.886	0.009					7	46	22	300	7.814	13.234	0.009	42	13.886	-1.266	0.009									7.814		
44	12.732	0.004	13.993	0.009	14.927	0.021			7	59	12	300	8.028	13.239	0.010	44	13.993	-1.261	0.010	-1.258	0.021			-0.934	0.023			8.028	0.003	
45	12.648	0.003	13.889	0.008	14.883	0.021			8	5	44	300	8.137	13.259	0.009	45	13.889	-1.241	0.009	-1.298	0.021			-0.994	0.022			8.137	-0.057	
46	12.574	0.003	13.803	0.008	14.766	0.019			8	11	22	300	8.231	13.271	0.009	46	13.803	-1.229	0.009	-1.255	0.019			-0.963	0.021			8.231	-0.026	
47	12.618	0.003	13.847	0.008	14.845	0.020			8	17	20	300	8.331	13.271	0.009	47	13.847	-1.229	0.009	-1.290	0.020			-0.998	0.022			8.331	-0.061	
48	12.711	0.003	13.940	0.007	14.934	0.018			8	23	0	300	8.425	13.271	0.008	48	13.940	-1.229	0.008	-1.286	0.018			-0.994	0.019			8.425	-0.057	
49	12.873	0.003	14.092	0.007	15.057	0.016			8	28	45	300	8.521	13.281	0.008	49	14.092	-1.219	0.008	-1.247	0.016			-0.965	0.017			8.521	-0.028	
50	12.818	0.003	14.045	0.007	14.988	0.018			8	34	25	300	8.615	13.273	0.008	50	14.045	-1.227	0.008	-1.233	0.018			-0.943	0.019			8.615	-0.006	
51	12.570	0.003	13.807	0.008					8	41	17	300	8.730	13.263	0.009	51	13.807	-1.237	0.009									8.730		
52	12.769	0.004	14.021	0.009	15.057	0.025			8	48	11	300	8.845	13.248	0.010	52	14.021	-1.252	0.010	-1.351	0.025			-1.036	0.027			8.845	-0.099	
53	12.762	0.003	14.002	0.007	14.987	0.016			8	54	10	300	8.944	13.260	0.008	53	14.002	-1.240	0.008	-1.288	0.016			-0.985	0.017			8.944	-0.048	
54	12.589	0.003	13.870	0.009					8	59	46	300	9.038	13.219	0.009	54	13.870	-1.281	0.009									9.038		
55	12.536	0.003	13.815	0.008					9	6	0	300	9.142	13.221	0.009	55	13.815	-1.279	0.009									9.142		
56	12.545	0.003	13.805	0.008					9	11	46	300	9.238	13.240	0.009	56	13.805	-1.260	0.009									9.238		
57	12.509	0.003	13.789	0.008					9	17	33	300	9.334	13.220	0.009	57	13.789	-1.280	0.009									9.334		
58	12.651	0.003	13.952	0.007	14.898	0.014			9	24	7	300	9.444	13.199	0.008	58	13.952	-1.301	0.008	-1.310	0.014			-0.946	0.016			9.444	-0.009	
59	12.514	0.003	13.796	0.008					9	29	49	300	9.539	13.218	0.009	59	13.796	-1.282	0.009									9.539		
60	12.506	0.003	13.801	0.008					9	35	37	300	9.635	13.205	0.009	60	13.801	-1.295	0.009									9.635		
61	12.513	0.003	13.814	0.008					9	41	15	300	9.729	13.199	0.009	61	13.814	-1.301	0.009									9.729		
62	12.519	0.003	13.830	0.008					9	48	4	300	9.843	13.189	0.009	62	13.830	-1.311	0.009									9.843		
63	12.562	0.003	13.879	0.009					9	53	56	300	9.941	13.183	0.009	63	13.879	-1.317	0.009									9.941		
64	12.566	0.003	13.916	0.009					10	0	0	300	10.042	13.150	0.009	64	13.916	-1.350	0.009									10.042		
66	12.543	0.003	13.886	0.009					10	13	5	300	10.260	13.157	0.009	66	13.886	-1.343	0.009									10.260		
mean	12.622		13.878		14.869		14.186							13.243				-1.257		-1.227		-1.274		-0.937		-0.364			0.000	0.000
sd	0.114		0.092		0.114		0.295							0.042				0.042		0.063		0.254		0.048		0.280			0.048	0.280
																										1				
																								Mags (SKY) ->		14.5				
																								Mags (Calc) ->						

583 Klotilde photometry (UT 98 Feb 22)												offset													
	|<--------				instrumental magnitudes						---------	11.529				|<-- differential photometry (adjusted) ----->							-1.643		
image	Klotilde		1		2						Time	comb		image		Klot-1		Klot-2		1-2		Time	1-2		
	mag	err	mag	err	mag	err	h	m	s	exp	(hours)	mag	err			mag	err	mag	err	mag	err	(hours)	mag		
1	12.709	0.003	11.678	0.002	13.311	0.004	3	19	0	300	3.358	12.560	0.004	1	11.678	1.031	0.004	1.041	0.005	-1.633	0.004	3.358	0.010		
2	12.704	0.003	11.675	0.002	13.308	0.004	3	25	41	300	3.470	12.558	0.004	2	11.675	1.029	0.004	1.039	0.005	-1.633	0.004	3.470	0.010		
3	12.721	0.003	11.683	0.002	13.326	0.004	3	31	38	300	3.569	12.567	0.004	3	11.683	1.038	0.004	1.038	0.005	-1.643	0.004	3.569	0.000		
4	12.706	0.003	11.678	0.002	13.318	0.004	3	37	30	300	3.667	12.557	0.004	4	11.678	1.028	0.004	1.031	0.005	-1.640	0.004	3.667	0.003		
5	12.696	0.003	11.670	0.002	13.304	0.004	3	43	9	300	3.761	12.555	0.004	5	11.670	1.026	0.004	1.035	0.005	-1.634	0.004	3.761	0.009		
6	12.696	0.003	11.664	0.002	13.301	0.004	3	48	50	300	3.856	12.561	0.004	6	11.664	1.032	0.004	1.038	0.005	-1.637	0.004	3.856	0.006		
7	12.698	0.003	11.663	0.002	13.297	0.004	3	54	27	300	3.949	12.564	0.004	7	11.663	1.035	0.004	1.044	0.005	-1.634	0.004	3.949	0.009		
8	12.713	0.003	11.673	0.002	13.313	0.004	4	0	0	300	4.042	12.569	0.004	8	11.673	1.040	0.004	1.043	0.005	-1.640	0.004	4.042	0.003		
9	12.718	0.003	11.670	0.002	13.304	0.004	4	5	33	300	4.134	12.577	0.004	9	11.670	1.048	0.004	1.057	0.005	-1.634	0.004	4.134	0.009		
10	12.721	0.003	11.665	0.002	13.303	0.004	4	11	18	300	4.230	12.585	0.004	10	11.665	1.056	0.004	1.061	0.005	-1.638	0.004	4.230	0.005		
11	12.729	0.003	11.663	0.002	13.301	0.004	4	16	48	300	4.322	12.595	0.004	11	11.663	1.066	0.004	1.071	0.005	-1.638	0.004	4.322	0.005		
12	12.722	0.003	11.656	0.002	13.292	0.004	4	22	40	300	4.419	12.595	0.004	12	11.656	1.066	0.004	1.073	0.005	-1.636	0.004	4.419	0.007		
13	12.721	0.003	11.657	0.002	13.298	0.004	4	29	42	300	4.537	12.593	0.004	13	11.657	1.064	0.004	1.066	0.005	-1.641	0.004	4.537	0.002		
14	12.707	0.003	11.654	0.002	13.288	0.004	4	35	16	300	4.629	12.582	0.004	14	11.654	1.053	0.004	1.062	0.005	-1.634	0.004	4.629	0.009		
15	12.697	0.003	11.651	0.002	13.289	0.004	4	40	55	300	4.724	12.575	0.004	15	11.651	1.046	0.004	1.051	0.005	-1.638	0.004	4.724	0.005		
16	12.692	0.003	11.650	0.002	13.283	0.004	4	46	30	300	4.817	12.571	0.004	16	11.650	1.042	0.004	1.052	0.005	-1.633	0.004	4.817	0.010		
17	12.688	0.003	11.655	0.002	13.289	0.004	4	52	10	300	4.911	12.562	0.004	17	11.655	1.033	0.004	1.042	0.005	-1.634	0.004	4.911	0.009		
18	12.679	0.003	11.648	0.002	13.282	0.004	4	57	46	300	5.004	12.560	0.004	18	11.648	1.031	0.004	1.040	0.005	-1.634	0.004	5.004	0.009		
19	12.679	0.003	11.655	0.002	13.289	0.004	5	3	29	300	5.100	12.553	0.004	19	11.655	1.024	0.004	1.033	0.005	-1.634	0.004	5.100	0.009		
20	12.664	0.003	11.650	0.002	13.287	0.004	5	9	8	300	5.194	12.543	0.004	20	11.650	1.014	0.004	1.020	0.005	-1.637	0.004	5.194	0.006		
21	12.668	0.003	11.658	0.002	13.288	0.004	5	14	52	300	5.289	12.539	0.004	21	11.658	1.010	0.004	1.023	0.005	-1.630	0.004	5.289	0.013		
22	12.666	0.003	11.659	0.002	13.296	0.004	5	20	38	300	5.386	12.536	0.004	22	11.659	1.007	0.004	1.013	0.005	-1.637	0.004	5.386	0.006		
23	12.656	0.003	11.657	0.002	13.299	0.004	5	27	23	300	5.498	12.528	0.004	23	11.657	0.999	0.004	1.000	0.005	-1.642	0.004	5.498	0.001		
25	12.649	0.003	11.659	0.002	13.299	0.004	5	38	51	300	5.689	12.519	0.004	25	11.659	0.990	0.004	0.993	0.005	-1.640	0.004	5.689	0.003		
26	12.641	0.003	11.656	0.002	13.293	0.004	5	45	0	300	5.792	12.514	0.004	26	11.656	0.985	0.004	0.991	0.005	-1.637	0.004	5.792	0.006		
27	12.641	0.003	11.659	0.002	13.301	0.004	5	50	41	300	5.886	12.511	0.004	27	11.659	0.982	0.004	0.983	0.005	-1.642	0.004	5.886	0.001		
28	12.637	0.003	11.645	0.002			6	16	7	300	6.310	12.521	0.004	28	11.645	0.992	0.004					6.310			
29	12.639	0.003	11.649	0.002			6	22	28	300	6.416	12.519	0.004	29	11.649	0.990	0.004					6.416			
30	12.654	0.003	11.647	0.002	13.284	0.005	6	28	32	300	6.517	12.536	0.004	30	11.647	1.007	0.004	1.013	0.006	-1.637	0.005	6.517	0.006		
31	12.663	0.003	11.646	0.002	13.281	0.004	6	34	23	300	6.615	12.546	0.004	31	11.646	1.017	0.004	1.025	0.005	-1.635	0.004	6.615	0.008		
32	12.681	0.003	11.648	0.002	13.284	0.004	6	40	13	300	6.712	12.562	0.004	32	11.648	1.033	0.004	1.040	0.005	-1.636	0.004	6.712	0.007		
33	12.699	0.003	11.652	0.002	13.296	0.004	6	46	13	300	6.812	12.576	0.004	33	11.652	1.047	0.004	1.046	0.005	-1.644	0.004	6.812	-0.001		
34	12.711	0.003	11.650	0.002	13.292	0.004	6	52	13	300	6.912	12.590	0.004	34	11.650	1.061	0.004	1.062	0.005	-1.642	0.004	6.912	0.001		
35	12.732	0.003	11.653	0.002	13.292	0.004	6	58	30	300	7.017	12.608	0.004	35	11.653	1.079	0.004	1.083	0.005	-1.639	0.004	7.017	0.004		
36	12.742	0.003	11.651	0.002	13.296	0.004	7	4	10	300	7.111	12.620	0.004	36	11.651	1.091	0.004	1.089	0.005	-1.645	0.004	7.111	-0.002		
37	12.765	0.003	11.659	0.002	13.293	0.004	7	9	48	300	7.205	12.635	0.004	37	11.659	1.106	0.004	1.115	0.005	-1.634	0.004	7.205	0.009		
38	12.776	0.003	11.659	0.002	13.297	0.004	7	15	26	300	7.299	12.646	0.004	38	11.659	1.117	0.004	1.122	0.005	-1.638	0.004	7.299	0.005		
39	12.791	0.003	11.669	0.002	13.303	0.004	7	21	9	300	7.394	12.651	0.004	39	11.669	1.122	0.004	1.131	0.005	-1.634	0.004	7.394	0.009		
40	12.792	0.003	11.661	0.002	13.304	0.004	7	26	47	300	7.488	12.660	0.004	40	11.661	1.131	0.004	1.131	0.005	-1.643	0.004	7.488	0.000		
41	12.800	0.003	11.666	0.002	13.306	0.004	7	32	34	300	7.584	12.663	0.004	41	11.666	1.134	0.004	1.137	0.005	-1.640	0.004	7.584	0.003		
42	12.800	0.003	11.663	0.002	13.304	0.004	7	38	25	300	7.682	12.666	0.004	42	11.663	1.137	0.004	1.139	0.005	-1.641	0.004	7.682	0.002		
43	12.802	0.003	11.666	0.002	13.303	0.004	7	44	2	300	7.776	12.665	0.004	43	11.666	1.136	0.004	1.142	0.005	-1.637	0.004	7.776	0.006		
44	12.803	0.003	11.668	0.002	13.305	0.004	7	49	43	300	7.870	12.664	0.004	44	11.668	1.135	0.004	1.141	0.005	-1.637	0.004	7.870	0.006		
45	12.812	0.003	11.679	0.002	13.316	0.004	7	55	27	300	7.966	12.662	0.004	45	11.679	1.133	0.004	1.139	0.005	-1.637	0.004	7.966	0.006		
46	12.837	0.003	11.709	0.002	13.350	0.004	8	1	15	300	8.063	12.657	0.004	46	11.709	1.128	0.004	1.130	0.005	-1.641	0.004	8.063	0.002		
47	13.080	0.004	11.955	0.002	13.606	0.005	8	10	7	300	8.210	12.654	0.004	47	11.955	1.125	0.004	1.117	0.006	-1.651	0.005	8.210	-0.008		
48	15.907	0.046	14.917	0.019			8	26	22	300	8.481	12.519	0.050	48	14.917	0.990	0.050					8.481			
49	15.982	0.049	14.894	0.018			8	32	5	300	8.576	12.617	0.052	49	14.894	1.088	0.052					8.576			
50	16.148	0.059	15.099	0.023			8	37	54	300	8.673	12.578	0.063	50	15.099	1.049	0.063					8.673			
51	15.129	0.019	14.096	0.008	15.927	0.039	8	44	3	300	8.776	12.562	0.021	51	14.096	1.033	0.021	0.845	0.043	-1.831	0.040	8.776	-0.188		
52	14.142	0.008	13.121	0.004	14.811	0.013	8	49	55	300	8.874	12.550	0.009	52	13.121	1.021	0.009	0.974	0.015	-1.690	0.014	8.874	-0.047		
53	13.995	0.008	12.982	0.004			8	55	48	300	8.972	12.542	0.009	53	12.982	1.013	0.009					8.972			
mean	13.010		11.958		13.396							12.581				1.052		1.056		-1.643			0.000		
sd	0.862		0.865		0.444							0.047				0.047		0.057		0.030			0.030		
																						1	2	1-2	
																				Mags (SKY) ->		11.4	13.3	-1.9	
																				Mags (Calc) ->		11.529	13.172	-1.643	-1.643
																								-0.257	

583 Klotilde photometry (UT 98 Mar 12)												offset												
	|<--------				instrumental magnitudes						---------	12.883				|<-- differential photometry (adjusted) ---->							-1.834	
image	Klotilde		1		2						Time	comb		image		Klot-1		Klot-2		1-2		Time	1-2	
	mag	err	mag	err	mag	err	h	m	s	exp	(hours)	mag	err			mag	err	mag	err	mag	err	(hours)	mag	
26	11.189	0.005	11.009	0.004	12.831	0.021	1	53	5	300	1.926	13.063	0.006	26	11.009	0.180	0.006	0.192	0.022	-1.822	0.021	1.926	0.012	
27	11.181	0.005	10.996	0.004	12.883	0.022	1	58	54	300	2.023	13.068	0.006	27	10.996	0.185	0.006	0.132	0.023	-1.887	0.022	2.023	-0.053	
34	11.223	0.004	10.996	0.003	12.847	0.016	2	32	27	300	2.583	13.110	0.005	34	10.996	0.227	0.005	0.210	0.016	-1.851	0.016	2.583	-0.017	
35	11.225	0.004	10.987	0.003	12.844	0.016	2	39	6	300	2.693	13.121	0.005	35	10.987	0.238	0.005	0.215	0.016	-1.857	0.016	2.693	-0.023	
36	11.248	0.004	10.992	0.003	12.826	0.015	2	45	8	300	2.794	13.139	0.005	36	10.992	0.256	0.005	0.256	0.016	-1.834	0.015	2.794	0.000	
37	11.190	0.005	10.969	0.004	12.800	0.019	4	8	37	300	4.185	13.104	0.006	37	10.969	0.221	0.006	0.224	0.020	-1.831	0.019	4.185	0.003	
38	11.201	0.005	10.981	0.004	12.806	0.019	4	15	28	300	4.299	13.103	0.006	38	10.981	0.220	0.006	0.229	0.020	-1.825	0.019	4.299	0.009	
39	11.192	0.004	10.991	0.004	12.844	0.019	4	21	33	300	4.401	13.084	0.006	39	10.991	0.201	0.006	0.182	0.019	-1.853	0.019	4.401	-0.019	
40	11.191	0.004	10.996	0.004	12.841	0.020	4	27	40	300	4.503	13.078	0.006	40	10.996	0.195	0.006	0.184	0.020	-1.845	0.020	4.503	-0.011	
41	11.190	0.004	10.996	0.004	12.830	0.020	4	34	23	300	4.615	13.077	0.006	41	10.996	0.194	0.006	0.194	0.020	-1.834	0.020	4.615	0.000	
42	11.172	0.004	10.992	0.004	12.822	0.020	4	40	31	300	4.717	13.063	0.006	42	10.992	0.180	0.006	0.184	0.020	-1.830	0.020	4.717	0.004	
43	11.163	0.004	10.988	0.004	12.835	0.019	4	46	34	300	4.818	13.058	0.006	43	10.988	0.175	0.006	0.162	0.019	-1.847	0.019	4.818	-0.013	
44	11.197	0.005	11.008	0.004	12.850	0.020	4	57	51	300	5.006	13.072	0.006	44	11.008	0.189	0.006	0.181	0.021	-1.842	0.020	5.006	-0.008	
45	11.183	0.004	11.004	0.004	12.809	0.019	5	3	34	300	5.101	13.062	0.006	45	11.004	0.179	0.006	0.208	0.019	-1.805	0.019	5.101	0.029	
46	11.199	0.004	10.992	0.004	12.826	0.019	5	9	8	300	5.194	13.090	0.006	46	10.992	0.207	0.006	0.207	0.019	-1.834	0.019	5.194	0.000	
47	11.188	0.004	10.986	0.004	12.848	0.020	5	14	49	300	5.289	13.085	0.006	47	10.986	0.202	0.006	0.174	0.020	-1.862	0.020	5.289	-0.028	
48	11.244	0.005	11.032	0.004	12.921	0.021	5	20	30	300	5.383	13.095	0.006	48	11.032	0.212	0.006	0.157	0.022	-1.889	0.021	5.383	-0.055	
49	11.247	0.005	11.014	0.004	12.843	0.020	5	37	1	300	5.659	13.116	0.006	49	11.014	0.233	0.006	0.238	0.021	-1.829	0.020	5.659	0.005	
50	11.269	0.005	11.016	0.004	12.896	0.022	5	43	28	300	5.766	13.136	0.006	50	11.016	0.253	0.006	0.207	0.023	-1.880	0.022	5.766	-0.046	
51	11.251	0.005	11.035	0.004	12.834	0.020	5	49	7	300	5.860	13.099	0.006	51	11.035	0.216	0.006	0.251	0.021	-1.799	0.020	5.860	0.035	
52	11.278	0.005	11.060	0.004	12.888	0.021	5	54	48	300	5.955	13.101	0.006	52	11.060	0.218	0.006	0.224	0.022	-1.828	0.021	5.955	0.006	
53	11.241	0.005	11.018	0.004	12.866	0.021	6	0	29	300	6.050	13.106	0.006	53	11.018	0.223	0.006	0.207	0.022	-1.850	0.021	6.050	-0.016	
54	11.230	0.005	11.022	0.004	12.853	0.021	6	6	22	300	6.148	13.091	0.006	54	11.022	0.208	0.006	0.211	0.022	-1.831	0.021	6.148	0.003	
55	11.231	0.005	11.031	0.004	12.888	0.021	6	14	11	300	6.278	13.083	0.006	55	11.031	0.200	0.006	0.177	0.022	-1.857	0.021	6.278	-0.023	
56	11.218	0.005	11.038	0.004	12.893	0.021	6	19	48	300	6.372	13.063	0.006	56	11.038	0.180	0.006	0.159	0.022	-1.855	0.021	6.372	-0.021	
57	11.259	0.004	11.083	0.004	12.972	0.024	6	25	21	300	6.464	13.059	0.006	57	11.083	0.176	0.006	0.121	0.024	-1.889	0.024	6.464	-0.055	
58	11.343	0.006	11.170	0.005	13.036	0.028	6	30	55	300	6.557	13.056	0.008	58	11.170	0.173	0.008	0.141	0.029	-1.866	0.028	6.557	-0.032	
59	11.220	0.005	11.050	0.004	12.909	0.022	6	36	32	300	6.651	13.053	0.006	59	11.050	0.170	0.006	0.145	0.023	-1.859	0.022	6.651	-0.025	
60	11.195	0.004	11.039	0.004	12.869	0.020	6	42	8	300	6.744	13.039	0.006	60	11.039	0.156	0.006	0.160	0.020	-1.830	0.020	6.744	0.004	
61	11.189	0.005	11.037	0.004	12.863	0.020	6	48	46	300	6.854	13.035	0.006	61	11.037	0.152	0.006	0.160	0.021	-1.826	0.020	6.854	0.008	
62	11.196	0.005	11.036	0.004	12.884	0.021	6	54	35	300	6.951	13.043	0.006	62	11.036	0.160	0.006	0.146	0.022	-1.848	0.021	6.951	-0.014	
63	11.206	0.005	11.040	0.004	12.895	0.021	7	0	17	300	7.046	13.049	0.006	63	11.040	0.166	0.006	0.145	0.022	-1.855	0.021	7.046	-0.021	
64	11.197	0.005	11.024	0.004	12.834	0.019	7	6	0	300	7.142	13.056	0.006	64	11.024	0.173	0.006	0.197	0.020	-1.810	0.019	7.142	0.024	
65	11.169	0.004	11.018	0.004	12.837	0.020	7	11	33	300	7.234	13.034	0.006	65	11.018	0.151	0.006	0.166	0.020	-1.819	0.020	7.234	0.015	
66	11.206	0.005	11.039	0.004	12.842	0.019	7	17	12	300	7.328	13.050	0.006	66	11.039	0.167	0.006	0.198	0.020	-1.803	0.019	7.328	0.031	
67	11.212	0.005	11.033	0.004	12.875	0.021	7	22	53	300	7.423	13.062	0.006	67	11.033	0.179	0.006	0.171	0.022	-1.842	0.021	7.423	-0.008	
68	11.246	0.005	11.061	0.004	12.895	0.021	7	28	33	300	7.518	13.068	0.006	68	11.061	0.185	0.006	0.185	0.022	-1.834	0.021	7.518	0.000	
69	11.254	0.005	11.053	0.004	12.846	0.020	7	34	9	300	7.611	13.084	0.006	69	11.053	0.201	0.006	0.242	0.021	-1.793	0.020	7.611	0.041	
70	11.270	0.005	11.046	0.004	12.812	0.020	7	39	45	300	7.704	13.107	0.006	70	11.046	0.224	0.006	0.292	0.021	-1.766	0.020	7.704	0.068	
71	11.282	0.005	11.054	0.004	12.853	0.020	7	45	30	300	7.800	13.111	0.006	71	11.054	0.228	0.006	0.263	0.021	-1.799	0.020	7.800	0.035	
72	11.320	0.005	11.091	0.004	12.858	0.021	7	51	9	300	7.894	13.112	0.006	72	11.091	0.229	0.006	0.296	0.022	-1.767	0.021	7.894	0.067	
73	11.474	0.006	11.215	0.005	13.029	0.025	7	56	48	300	7.988	13.142	0.008	73	11.215	0.259	0.008	0.279	0.026	-1.814	0.025	7.988	0.020	
74	11.450	0.007	11.208	0.005	12.925	0.026	8	4	3	300	8.109	13.125	0.009	74	11.208	0.242	0.009	0.359	0.027	-1.717	0.026	8.109	0.117	
75	11.551	0.009	11.281	0.007	13.133	0.036	8	9	40	300	8.203	13.153	0.011	75	11.281	0.270	0.011	0.252	0.037	-1.852	0.037	8.203	-0.018	
76	11.366	0.006	11.067	0.004	12.953	0.025	8	15	25	300	8.299	13.182	0.007	76	11.067	0.299	0.007	0.247	0.026	-1.886	0.025	8.299	-0.052	
77	11.372	0.006	11.076	0.004	12.839	0.022	8	22	13	300	8.412	13.179	0.007	77	11.076	0.296	0.007	0.367	0.023	-1.763	0.022	8.412	0.071	
78	11.393	0.006	11.089	0.005	12.885	0.024	8	27	51	300	8.506	13.187	0.008	78	11.089	0.304	0.008	0.342	0.025	-1.796	0.025	8.506	0.038	
79	11.418	0.006	11.107	0.005	12.900	0.025	8	33	30	300	8.600	13.194	0.008	79	11.107	0.311	0.008	0.352	0.026	-1.793	0.025	8.600	0.041	
80	11.446	0.007	11.116	0.005	13.061	0.030	8	39	12	300	8.695	13.213	0.009	80	11.116	0.330	0.009	0.219	0.031	-1.945	0.030	8.695	-0.111	
81	11.457	0.007	11.148	0.005	12.998	0.028	8	45	16	300	8.796	13.192	0.009	81	11.148	0.309	0.009	0.293	0.029	-1.850	0.028	8.796	-0.016	
82	11.455	0.007	11.141	0.005	12.952	0.028	8	51	5	300	8.893	13.197	0.009	82	11.141	0.314	0.009	0.337	0.029	-1.811	0.028	8.893	0.023	
83	11.500	0.007	11.172	0.005	13.114	0.033	8	56	44	300	8.987	13.211	0.009	83	11.172	0.328	0.009	0.220	0.034	-1.942	0.033	8.987	-0.108	
84	11.497	0.007	11.176	0.006	12.972	0.029	9	2	41	300	9.086	13.204	0.009	84	11.176	0.321	0.009	0.359	0.030	-1.796	0.030	9.086	0.038	
85	11.528	0.008	11.205	0.006	13.097	0.034	9	8	24	300	9.182	13.206	0.010	85	11.205	0.323	0.010	0.265	0.035	-1.892	0.035	9.182	-0.058	
86	11.523	0.008	11.224	0.006	12.972	0.031	9	14	32	300	9.284	13.182	0.010	86	11.224	0.299	0.010	0.385	0.032	-1.748	0.032	9.284	0.086	
87	11.548	0.008	11.299	0.007	13.141	0.038	9	20	20	300	9.381	13.132	0.011	87	11.299	0.249	0.011	0.241	0.039	-1.842	0.039	9.381	-0.008	
mean	11.287		11.063		12.898							13.107				0.224		0.224		**-1.834**			0.000	
sd	0.118		0.080		0.086							0.052				0.052		0.066		0.042			0.042	
																						1	2	
																				Mags (SKY) ->		12.8	14.8	-2
																				Mags (Calc) ->		12.883	14.717	-1.834
																								-0.166

583 Klotilde photometry (UT 98 Mar 13)

														offset													
	<-----			instrumental magnitudes						------>				11.722				<-- differential photometry (adjusted) ---->							0.444		
image	Klotilde		1		2		3						Time	comb		image		Klot-1		Klot-2		1-2		Time	1-2		
	mag	err	mag	err	mag	err	mag	err	h	m	s	exp	(hours)	mag	err			mag	err	mag	err	mag	err	(hours)	mag		
1	11.573	0.005	10.265	0.002	9.790	0.001	13.216	0.024	2	47	19	300	2.830	13.030	0.005	1	10.265	1.308	0.005	1.339	0.005	0.475	0.002	2.830	0.031		
2	11.363	0.005	10.042	0.002	9.570	0.001	12.960	0.021	2	53	39	300	2.936	13.043	0.005	2	10.042	1.321	0.005	1.349	0.005	0.472	0.002	2.936	0.028		
3	11.310	0.005	9.983	0.002	9.506	0.001	12.933	0.022	2	59	26	300	3.032	13.049	0.005	3	9.983	1.327	0.005	1.360	0.005	0.477	0.002	3.032	0.033		
4	11.313	0.005	9.990	0.002	9.515	0.001	12.956	0.021	3	6	26	300	3.149	13.045	0.005	4	9.990	1.323	0.005	1.354	0.005	0.475	0.002	3.149	0.031		
5	11.325	0.005	9.989	0.002	9.512	0.001	12.984	0.021	3	12	27	300	3.249	13.058	0.005	5	9.989	1.336	0.005	1.369	0.005	0.477	0.002	3.249	0.033		
6	11.322	0.005	9.978	0.002	9.507	0.001	12.861	0.019	3	18	0	300	3.342	13.066	0.005	6	9.978	1.344	0.005	1.371	0.005	0.471	0.002	3.342	0.027		
7	11.325	0.005	9.975	0.002	9.514	0.001	12.952	0.021	3	23	38	300	3.436	13.072	0.005	7	9.975	1.350	0.005	1.367	0.005	0.461	0.002	3.436	0.017		
8	11.319	0.005	9.976	0.002	9.522	0.001	12.934	0.021	3	29	27	300	3.533	13.065	0.005	8	9.976	1.343	0.005	1.353	0.005	0.454	0.002	3.533	0.010		
9	11.300	0.005	9.970	0.002	9.525	0.001	12.938	0.020	3	35	3	300	3.626	13.052	0.005	9	9.970	1.330	0.005	1.331	0.005	0.445	0.002	3.626	0.001		
10	11.300	0.005	9.973	0.001	9.529	0.001	12.957	0.020	3	40	54	300	3.723	13.049	0.005	10	9.973	1.327	0.005	1.327	0.005	0.444	0.001	3.723	0.000		
11	11.289	0.004	9.972	0.001	9.536	0.001	12.903	0.019	3	46	31	300	3.817	13.039	0.004	11	9.972	1.317	0.004	1.309	0.004	0.436	0.001	3.817	-0.008		
12	11.273	0.004	9.970	0.001	9.544	0.001	12.933	0.019	3	52	11	300	3.911	13.025	0.004	12	9.970	1.303	0.004	1.285	0.004	0.426	0.001	3.911	-0.018		
13	11.263	0.004	9.971	0.001	9.548	0.001	12.928	0.019	3	57	48	300	4.005	13.014	0.004	13	9.971	1.292	0.004	1.271	0.004	0.423	0.001	4.005	-0.021		
14	11.246	0.004	9.976	0.001	9.554	0.001	12.929	0.019	4	3	26	300	4.099	12.992	0.004	14	9.976	1.270	0.004	1.248	0.004	0.422	0.001	4.099	-0.022		
15	11.233	0.004	9.973	0.001	9.553	0.001	12.945	0.019	4	9	3	300	4.193	12.982	0.004	15	9.973	1.260	0.004	1.236	0.004	0.420	0.001	4.193	-0.024		
16	11.216	0.004	9.968	0.001	9.552	0.001	12.905	0.018	4	14	40	300	4.286	12.970	0.004	16	9.968	1.248	0.004	1.220	0.004	0.416	0.001	4.286	-0.028		
17	11.199	0.004	9.968	0.001	9.550	0.001	12.895	0.018	4	20	14	300	4.379	12.953	0.004	17	9.968	1.231	0.004	1.205	0.004	0.418	0.001	4.379	-0.026		
18	11.203	0.004	9.966	0.001	9.543	0.001	12.903	0.019	4	25	53	300	4.473	12.959	0.004	18	9.966	1.237	0.004	1.216	0.004	0.423	0.001	4.473	-0.021		
19	11.188	0.004	9.968	0.002	9.541	0.001	12.955	0.021	4	31	29	300	4.566	12.942	0.004	19	9.968	1.220	0.004	1.203	0.004	0.427	0.002	4.566	-0.017		
20	11.184	0.004	9.967	0.002	9.532	0.001	12.906	0.020	4	37	4	300	4.659	12.939	0.004	20	9.967	1.217	0.004	1.208	0.004	0.435	0.002	4.659	-0.009		
21	11.179	0.004	9.967	0.001	9.521	0.001	12.913	0.020	4	42	41	300	4.753	12.934	0.004	21	9.967	1.212	0.004	1.214	0.004	0.446	0.001	4.753	0.002		
22	11.177	0.004	9.968	0.002	9.512	0.001	12.905	0.021	4	48	20	300	4.847	12.931	0.004	22	9.968	1.209	0.004	1.221	0.004	0.456	0.002	4.847	0.012		
23	11.168	0.005	9.973	0.002	9.509	0.001	12.946	0.022	4	54	4	300	4.943	12.917	0.005	23	9.973	1.195	0.005	1.215	0.005	0.464	0.002	4.943	0.020		
24	11.163	0.004	9.972	0.002	9.501	0.001	12.909	0.020	4	59	38	300	5.036	12.913	0.004	24	9.972	1.191	0.004	1.218	0.004	0.471	0.002	5.036	0.027		
25	11.172	0.004	9.968	0.002	9.499	0.001	12.912	0.021	5	5	28	300	5.133	12.926	0.004	25	9.968	1.204	0.004	1.229	0.004	0.469	0.002	5.133	0.025		
26	11.161	0.005	9.966	0.002	9.494	0.001	12.911	0.023	5	11	5	300	5.226	12.917	0.005	26	9.966	1.195	0.005	1.223	0.005	0.472	0.002	5.226	0.028		
27	11.166	0.005	9.965	0.002	9.497	0.001	12.976	0.024	5	16	45	300	5.321	12.923	0.005	27	9.965	1.201	0.005	1.225	0.005	0.468	0.002	5.321	0.024		
28	11.159	0.005	9.961	0.002	9.505	0.001	12.872	0.022	5	22	23	300	5.415	12.920	0.005	28	9.961	1.198	0.005	1.210	0.005	0.456	0.002	5.415	0.012		
29	11.157	0.005	9.970	0.002	9.518	0.001	12.929	0.022	5	29	30	300	5.533	12.909	0.005	29	9.970	1.187	0.005	1.195	0.005	0.452	0.002	5.533	0.008		
30	11.167	0.005	9.971	0.002	9.530	0.001	12.922	0.023	5	35	17	300	5.630	12.918	0.005	30	9.971	1.196	0.005	1.193	0.005	0.441	0.002	5.630	-0.003		
31	11.187	0.005	9.975	0.002	9.539	0.001	12.918	0.021	5	40	55	300	5.724	12.934	0.005	31	9.975	1.212	0.005	1.204	0.005	0.436	0.002	5.724	-0.008		
32	11.203	0.005	9.976	0.002	9.548	0.001	12.956	0.023	5	46	30	300	5.817	12.949	0.005	32	9.976	1.227	0.005	1.211	0.005	0.428	0.002	5.817	-0.016		
33	11.211	0.005	9.977	0.002	9.556	0.001	12.985	0.024	5	52	7	300	5.910	12.956	0.005	33	9.977	1.234	0.005	1.211	0.005	0.421	0.002	5.910	-0.023		
34	11.231	0.005	9.974	0.002	9.560	0.001	12.945	0.023	5	57	42	300	6.003	12.979	0.005	34	9.974	1.257	0.005	1.227	0.005	0.414	0.002	6.003	-0.030		
35	11.240	0.005	9.980	0.002	9.570	0.001	12.932	0.022	6	3	20	300	6.097	12.982	0.005	35	9.980	1.260	0.005	1.226	0.005	0.410	0.002	6.097	-0.034		
36	11.250	0.005	9.988	0.002	9.576	0.001	12.981	0.023	6	9	1	300	6.192	12.984	0.005	36	9.988	1.262	0.005	1.230	0.005	0.412	0.002	6.192	-0.032		
37	11.250	0.005	9.999	0.002	9.578	0.001	13.006	0.024	6	15	9	300	6.294	12.973	0.005	37	9.999	1.251	0.005	1.228	0.005	0.421	0.002	6.294	-0.022		
38	11.259	0.005	9.996	0.002	9.580	0.001	13.009	0.024	6	21	22	300	6.398	12.985	0.005	38	9.996	1.263	0.005	1.235	0.005	0.416	0.002	6.398	-0.028		
39	11.284	0.005	9.996	0.002	9.580	0.001	13.048	0.025	6	26	58	300	6.491	13.010	0.005	39	9.996	1.288	0.005	1.260	0.005	0.416	0.002	6.491	-0.028		
40	11.295	0.005	9.996	0.002	9.579	0.001	13.071	0.026	6	32	42	300	6.587	13.021	0.005	40	9.996	1.299	0.005	1.272	0.005	0.417	0.002	6.587	-0.027		
41	11.311	0.005	10.000	0.002	9.573	0.001	13.027	0.025	6	38	26	300	6.682	13.033	0.005	41	10.000	1.311	0.005	1.294	0.005	0.427	0.002	6.682	-0.017		
42	11.324	0.005	10.009	0.002	9.576	0.001	13.036	0.025	6	44	5	300	6.776	13.037	0.005	42	10.009	1.315	0.005	1.304	0.005	0.433	0.002	6.776	-0.011		
43	11.301	0.005	10.002	0.002	9.562	0.001	13.052	0.026	6	49	45	300	6.871	13.021	0.005	43	10.002	1.299	0.005	1.295	0.005	0.440	0.002	6.871	-0.004		
44	11.321	0.004	10.023	0.001	9.568	0.001	12.971	0.019	6	55	20	300	6.964	13.020	0.004	44	10.023	1.298	0.004	1.309	0.004	0.455	0.001	6.964	0.011		
45	11.312	0.004	10.016	0.001	9.561	0.001	12.994	0.019	7	0	56	300	7.057	13.018	0.004	45	10.016	1.296	0.004	1.307	0.004	0.455	0.001	7.057	0.011		
46	11.327	0.004	10.024	0.001	9.560	0.001	13.009	0.020	7	6	36	300	7.152	13.025	0.004	46	10.024	1.303	0.004	1.323	0.004	0.464	0.001	7.152	0.020		
47	11.328	0.004	10.029	0.001	9.567	0.001	13.019	0.020	7	12	26	300	7.249	13.021	0.004	47	10.029	1.299	0.004	1.317	0.004	0.462	0.001	7.249	0.018		
48	11.329	0.004	10.039	0.001	9.579	0.001	12.985	0.020	7	18	4	300	7.343	13.012	0.004	48	10.039	1.290	0.004	1.306	0.004	0.460	0.001	7.343	0.016		
49	11.310	0.004	10.029	0.001	9.579	0.001	12.945	0.019	7	23	43	300	7.437	13.003	0.004	49	10.029	1.281	0.004	1.287	0.004	0.450	0.001	7.437	0.006		
50	11.335	0.004	10.060	0.001	9.610	0.001	12.985	0.017	7	29	30	300	7.533	12.997	0.004	50	10.060	1.275	0.004	1.281	0.004	0.450	0.001	7.533	0.006		
mean	11.260		9.992		9.549									12.990				1.268		1.268		0.444			0.000		
sd	0.077		0.046		0.045									0.049				0.049		0.056		0.021			0.021		
																								1	2	3	1-2
																						Mags (SKY) ->		11.8	11.2	14.9	0.6
																						Mags (Calc) ->		11.722	11.278		0.444
																											0.156

Combined Observations

583 Klotilde						period=	9.211	0.3072			13-Feb	22-Feb	12-Mar	13-Mar
											0	0.7	0.3	0.45
	Raw	Seq.			Sun	Earth	0.071				New	New	New	New
Date	Time	Time	Klotilde's		Distance	Distance					Klotilde	Klotilde	Klotilde	Klotilde
	(hours)	(hours)	Mag	Mag err	(a.u.)	(a.u)	Phase				Mag.	Mag.	Mag.	Mag.
13-Feb	4.763	4.763	13.198	0.009	2.675	1.709	0.562	9.897	0.009	0.562	9.897	0.000	0.000	0.000
	4.860	4.860	13.188	0.009	2.675	1.709	0.573	9.887	0.009	0.573	9.887	0.000	0.000	0.000
	5.062	5.062	13.200	0.009	2.675	1.709	0.595	9.899	0.009	0.595	9.899	0.000	0.000	0.000
	5.178	5.178	13.192	0.009	2.675	1.709	0.607	9.891	0.009	0.607	9.891	0.000	0.000	0.000
	5.366	5.366	13.196	0.009	2.675	1.709	0.628	9.895	0.009	0.628	9.895	0.000	0.000	0.000
	5.469	5.469	13.201	0.009	2.675	1.709	0.639	9.900	0.009	0.639	9.900	0.000	0.000	0.000
	5.581	5.581	13.218	0.009	2.675	1.709	0.651	9.917	0.009	0.651	9.917	0.000	0.000	0.000
	5.775	5.775	13.233	0.009	2.675	1.709	0.672	9.932	0.009	0.672	9.932	0.000	0.000	0.000
	5.910	5.910	13.281	0.008	2.675	1.709	0.687	9.980	0.008	0.687	9.980	0.000	0.000	0.000
	6.021	6.021	13.279	0.008	2.675	1.709	0.699	9.978	0.008	0.699	9.978	0.000	0.000	0.000
	6.144	6.144	13.303	0.008	2.675	1.709	0.712	10.002	0.008	0.712	10.002	0.000	0.000	0.000
	6.408	6.408	13.298	0.011	2.675	1.709	0.741	9.997	0.011	0.741	9.997	0.000	0.000	0.000
	6.511	6.511	13.334	0.013	2.675	1.709	0.752	10.033	0.013	0.752	10.033	0.000	0.000	0.000
	6.610	6.610	13.294	0.009	2.675	1.709	0.763	9.993	0.009	0.763	9.993	0.000	0.000	0.000
	6.725	6.725	13.295	0.011	2.675	1.709	0.775	9.994	0.011	0.775	9.994	0.000	0.000	0.000
	6.825	6.825	13.301	0.011	2.675	1.709	0.786	10.000	0.011	0.786	10.000	0.000	0.000	0.000
	6.939	6.939	13.277	0.010	2.675	1.709	0.799	9.976	0.010	0.799	9.976	0.000	0.000	0.000
	7.040	7.040	13.284	0.009	2.675	1.709	0.810	9.983	0.009	0.810	9.983	0.000	0.000	0.000
	7.137	7.137	13.288	0.009	2.675	1.709	0.820	9.987	0.009	0.820	9.987	0.000	0.000	0.000
	7.233	7.233	13.276	0.009	2.675	1.709	0.831	9.975	0.009	0.831	9.975	0.000	0.000	0.000
	7.344	7.344	13.263	0.008	2.675	1.709	0.843	9.962	0.008	0.843	9.962	0.000	0.000	0.000
	7.440	7.440	13.257	0.008	2.675	1.709	0.853	9.956	0.008	0.853	9.956	0.000	0.000	0.000
	7.533	7.533	13.259	0.008	2.675	1.709	0.863	9.958	0.008	0.863	9.958	0.000	0.000	0.000
	7.628	7.628	13.255	0.009	2.675	1.709	0.873	9.954	0.009	0.873	9.954	0.000	0.000	0.000
	7.721	7.721	13.240	0.009	2.675	1.709	0.884	9.939	0.009	0.884	9.939	0.000	0.000	0.000
	7.814	7.814	13.234	0.009	2.675	1.709	0.894	9.933	0.009	0.894	9.933	0.000	0.000	0.000
	8.028	8.028	13.239	0.010	2.675	1.709	0.917	9.938	0.010	0.917	9.938	0.000	0.000	0.000
	8.137	8.137	13.259	0.009	2.675	1.709	0.929	9.958	0.009	0.929	9.958	0.000	0.000	0.000
	8.231	8.231	13.271	0.009	2.675	1.709	0.939	9.970	0.009	0.939	9.970	0.000	0.000	0.000
	8.331	8.331	13.271	0.009	2.675	1.709	0.950	9.970	0.009	0.950	9.970	0.000	0.000	0.000
	8.425	8.425	13.271	0.008	2.675	1.709	0.960	9.970	0.008	0.960	9.970	0.000	0.000	0.000

Combined Observations

	8.521	8.521	13.281	0.008	2.675	1.709	0.970	9.980	0.008	0.970	9.980	0.000	0.000	0.000	
	8.615	8.615	13.273	0.008	2.675	1.709	0.981	9.972	0.008	0.981	9.972	0.000	0.000	0.000	
	8.730	8.730	13.263	0.009	2.675	1.709	0.993	9.962	0.009	0.993	9.962	0.000	0.000	0.000	
	8.845	8.845	13.248	0.010	2.675	1.709	0.006	9.947	0.010	0.006	9.947	0.000	0.000	0.000	
	8.944	8.944	13.260	0.008	2.675	1.709	0.016	9.959	0.008	0.016	9.959	0.000	0.000	0.000	
	9.038	9.038	13.219	0.009	2.675	1.709	0.026	9.918	0.009	0.026	9.918	0.000	0.000	0.000	
	9.142	9.142	13.221	0.009	2.675	1.709	0.038	9.920	0.009	0.038	9.920	0.000	0.000	0.000	
	9.238	9.238	13.240	0.009	2.675	1.709	0.048	9.939	0.009	0.048	9.939	0.000	0.000	0.000	
	9.334	9.334	13.220	0.009	2.675	1.709	0.059	9.919	0.009	0.059	9.919	0.000	0.000	0.000	
	9.444	9.444	13.199	0.008	2.675	1.709	0.071	9.898	0.008	0.071	9.898	0.000	0.000	0.000	
	9.539	9.539	13.218	0.009	2.675	1.709	0.081	9.917	0.009	0.081	9.917	0.000	0.000	0.000	
	9.635	9.635	13.205	0.009	2.675	1.709	0.091	9.904	0.009	0.091	9.904	0.000	0.000	0.000	
	9.729	9.729	13.199	0.009	2.675	1.709	0.102	9.898	0.009	0.102	9.898	0.000	0.000	0.000	
	9.843	9.843	13.189	0.009	2.675	1.709	0.114	9.888	0.009	0.114	9.888	0.000	0.000	0.000	
	9.941	9.941	13.183	0.009	2.675	1.709	0.124	9.882	0.009	0.124	9.882	0.000	0.000	0.000	
	10.042	10.042	13.150	0.009	2.675	1.709	0.135	9.849	0.009	0.135	9.849	0.000	0.000	0.000	
	10.260	10.260	13.157	0.009	2.675	1.709	0.159	9.856	0.009	0.159	9.856	0.000	0.000	0.000	
22-Feb	3.358	219.358	12.560	0.004	2.672	1.736	0.860	0.000	0.004	0.860		9.928	0.000	0.000	9
	3.470	219.470	12.558	0.004	2.672	1.736	0.872	0.000	0.004	0.872		9.926	0.000	0.000	10
	3.569	219.569	12.567	0.004	2.672	1.736	0.883	0.000	0.004	0.883		9.935	0.000	0.000	11
	3.667	219.667	12.557	0.004	2.672	1.736	0.893	0.000	0.004	0.893		9.925	0.000	0.000	12
	3.761	219.761	12.555	0.004	2.672	1.736	0.903	0.000	0.004	0.903		9.923	0.000	0.000	13
	3.856	219.856	12.561	0.004	2.672	1.736	0.914	0.000	0.004	0.914		9.929	0.000	0.000	14
	3.949	219.949	12.564	0.004	2.672	1.736	0.924	0.000	0.004	0.924		9.932	0.000	0.000	15
	4.042	220.042	12.569	0.004	2.672	1.736	0.934	0.000	0.004	0.934		9.937	0.000	0.000	16
	4.134	220.134	12.577	0.004	2.672	1.736	0.944	0.000	0.004	0.944		9.945	0.000	0.000	17
	4.230	220.230	12.585	0.004	2.672	1.736	0.954	0.000	0.004	0.954		9.953	0.000	0.000	18
	4.322	220.322	12.595	0.004	2.672	1.736	0.964	0.000	0.004	0.964		9.963	0.000	0.000	19
	4.419	220.419	12.595	0.004	2.672	1.736	0.975	0.000	0.004	0.975		9.963	0.000	0.000	20
	4.537	220.537	12.593	0.004	2.672	1.736	0.988	0.000	0.004	0.988		9.961	0.000	0.000	21
	4.629	220.629	12.582	0.004	2.672	1.736	0.998	0.000	0.004	0.998		9.950	0.000	0.000	22
	4.724	220.724	12.575	0.004	2.672	1.736	0.008	0.000	0.004	0.008		9.943	0.000	0.000	23
	4.817	220.817	12.571	0.004	2.672	1.736	0.018	0.000	0.004	0.018		9.939	0.000	0.000	24
	4.911	220.911	12.562	0.004	2.672	1.736	0.028	0.000	0.004	0.028		9.930	0.000	0.000	25
	5.004	221.004	12.560	0.004	2.672	1.736	0.038	0.000	0.004	0.038		9.928	0.000	0.000	26
	5.100	221.100	12.553	0.004	2.672	1.736	0.049	0.000	0.004	0.049		9.921	0.000	0.000	27
	5.194	221.194	12.543	0.004	2.672	1.736	0.059	0.000	0.004	0.059		9.911	0.000	0.000	

Combined Observations

	5.289	221.289	12.539	0.004	2.672	1.736	0.069	0.000	0.004	0.069		9.907	0.000	0.000
	5.386	221.386	12.536	0.004	2.672	1.736	0.080	0.000	0.004	0.080		9.904	0.000	0.000
	5.498	221.498	12.528	0.004	2.672	1.736	0.092	0.000	0.004	0.092		9.896	0.000	0.000
	5.689	221.689	12.519	0.004	2.672	1.736	0.113	0.000	0.004	0.113		9.887	0.000	0.000
	5.792	221.792	12.514	0.004	2.672	1.736	0.124	0.000	0.004	0.124		9.882	0.000	0.000
	5.886	221.886	12.511	0.004	2.672	1.736	0.134	0.000	0.004	0.134		9.879	0.000	0.000
	6.310	222.310	12.521	0.004	2.672	1.736	0.180	0.000	0.004	0.180		9.889	0.000	0.000
	6.416	222.416	12.519	0.004	2.672	1.736	0.192	0.000	0.004	0.192		9.887	0.000	0.000
	6.517	222.517	12.536	0.004	2.672	1.736	0.203	0.000	0.004	0.203		9.904	0.000	0.000
	6.615	222.615	12.546	0.004	2.672	1.736	0.213	0.000	0.004	0.213		9.914	0.000	0.000
	6.712	222.712	12.562	0.004	2.672	1.736	0.224	0.000	0.004	0.224		9.930	0.000	0.000
	6.812	222.812	12.576	0.004	2.672	1.736	0.235	0.000	0.004	0.235		9.944	0.000	0.000
	6.912	222.912	12.590	0.004	2.672	1.736	0.246	0.000	0.004	0.246		9.958	0.000	0.000
	7.017	223.017	12.608	0.004	2.672	1.736	0.257	0.000	0.004	0.257		9.976	0.000	0.000
	7.111	223.111	12.620	0.004	2.672	1.736	0.267	0.000	0.004	0.267		9.988	0.000	0.000
	7.205	223.205	12.635	0.004	2.672	1.736	0.277	0.000	0.004	0.277		10.003	0.000	0.000
	7.299	223.299	12.646	0.004	2.672	1.736	0.288	0.000	0.004	0.288		10.014	0.000	0.000
	7.394	223.394	12.651	0.004	2.672	1.736	0.298	0.000	0.004	0.298		10.019	0.000	0.000
	7.488	223.488	12.660	0.004	2.672	1.736	0.308	0.000	0.004	0.308		10.028	0.000	0.000
	7.584	223.584	12.663	0.004	2.672	1.736	0.319	0.000	0.004	0.319		10.031	0.000	0.000
	7.682	223.682	12.666	0.004	2.672	1.736	0.329	0.000	0.004	0.329		10.034	0.000	0.000
	7.776	223.776	12.665	0.004	2.672	1.736	0.339	0.000	0.004	0.339		10.033	0.000	0.000
	7.870	223.870	12.664	0.004	2.672	1.736	0.350	0.000	0.004	0.350		10.032	0.000	0.000
	7.966	223.966	12.662	0.004	2.672	1.736	0.360	0.000	0.004	0.360		10.030	0.000	0.000
	8.063	224.063	12.657	0.004	2.672	1.736	0.370	0.000	0.004	0.370		10.025	0.000	0.000
	8.210	224.210	12.654	0.004	2.672	1.736	0.386	0.000	0.004	0.386		10.022	0.000	0.000
	8.481	224.481	12.519	0.050	2.672	1.736	0.416	0.000	0.050	0.416		9.887	0.000	0.000
	8.576	224.576	12.617	0.052	2.672	1.736	0.426	0.000	0.052	0.426		9.985	0.000	0.000
	8.673	224.673	12.578	0.063	2.672	1.736	0.437	0.000	0.063	0.437		9.946	0.000	0.000
	8.776	224.776	12.562	0.021	2.672	1.736	0.448	0.000	0.021	0.448		9.930	0.000	0.000
	8.874	224.874	12.550	0.009	2.672	1.736	0.458	0.000	0.009	0.458		9.918	0.000	0.000
	8.972	224.972	12.542	0.009	2.672	1.736	0.469	0.000	0.009	0.469		9.910	0.000	0.000
12-Mar	1.926	649.926	13.063	0.006	2.6666	1.8485	0.603	0.000	0.006	0.603			9.899	0.000
	2.023	650.023	13.068	0.006	2.6666	1.8485	0.614	0.000	0.006	0.614			9.904	0.000
	2.583	650.583	13.110	0.005	2.6666	1.8485	0.674	0.000	0.005	0.674			9.946	0.000
	2.693	650.693	13.121	0.005	2.6666	1.8485	0.686	0.000	0.005	0.686			9.957	0.000
	2.794	650.794	13.139	0.005	2.6666	1.8485	0.697	0.000	0.005	0.697			9.975	0.000

Combined Observations

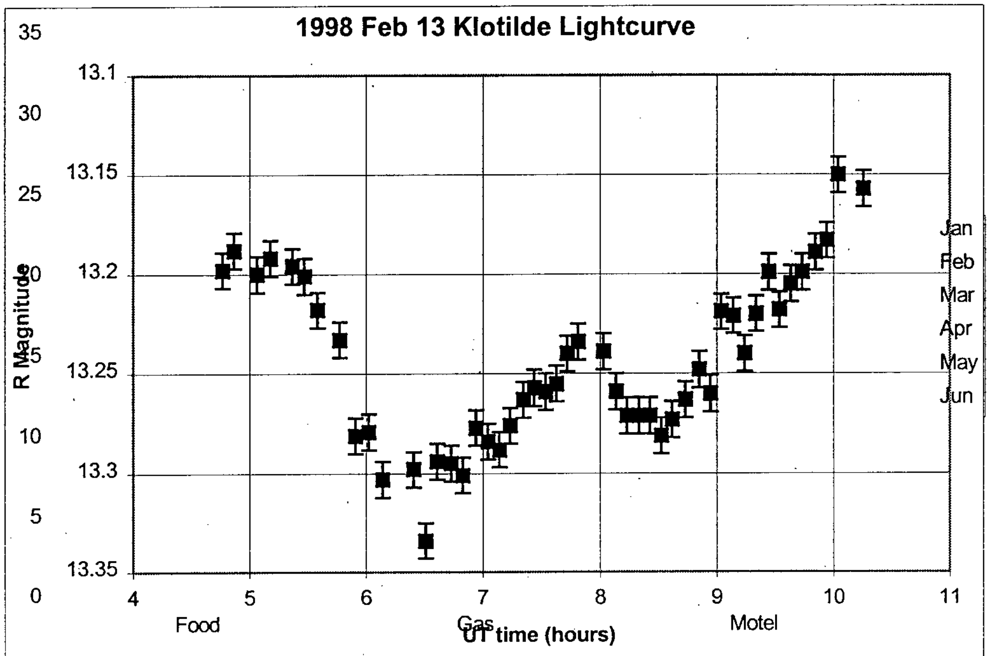
	4.185	652.185	13.104	0.006	2.6666	1.8485	0.848	0.000	0.006	0.848			9.940	0.000
	4.299	652.299	13.103	0.006	2.6666	1.8485	0.861	0.000	0.006	0.861			9.939	0.000
	4.401	652.401	13.084	0.006	2.6666	1.8485	0.872	0.000	0.006	0.872			9.920	0.000
	4.503	652.503	13.078	0.006	2.6666	1.8485	0.883	0.000	0.006	0.883			9.914	0.000
	4.615	652.615	13.077	0.006	2.6666	1.8485	0.895	0.000	0.006	0.895			9.913	0.000
	4.717	652.717	13.063	0.006	2.6666	1.8485	0.906	0.000	0.006	0.906			9.899	0.000
	4.818	652.818	13.058	0.006	2.6666	1.8485	0.917	0.000	0.006	0.917			9.894	0.000
	5.006	653.006	13.072	0.006	2.6666	1.8485	0.937	0.000	0.006	0.937			9.908	0.000
	5.101	653.101	13.062	0.006	2.6666	1.8485	0.948	0.000	0.006	0.948			9.898	0.000
	5.194	653.194	13.090	0.006	2.6666	1.8485	0.958	0.000	0.006	0.958			9.926	0.000
	5.289	653.289	13.085	0.006	2.6666	1.8485	0.968	0.000	0.006	0.968			9.921	0.000
	5.383	653.383	13.095	0.006	2.6666	1.8485	0.978	0.000	0.006	0.978			9.931	0.000
	5.659	653.659	13.116	0.006	2.6666	1.8485	0.008	0.000	0.006	0.008			9.952	0.000
	5.766	653.766	13.136	0.006	2.6666	1.8485	0.020	0.000	0.006	0.020			9.972	0.000
	5.860	653.860	13.099	0.006	2.6666	1.8485	0.030	0.000	0.006	0.030			9.935	0.000
	5.955	653.955	13.101	0.006	2.6666	1.8485	0.040	0.000	0.006	0.040			9.937	0.000
	6.050	654.050	13.106	0.006	2.6666	1.8485	0.051	0.000	0.006	0.051			9.942	0.000
	6.148	654.148	13.091	0.006	2.6666	1.8485	0.061	0.000	0.006	0.061			9.927	0.000
	6.278	654.278	13.083	0.006	2.6666	1.8485	0.075	0.000	0.006	0.075			9.919	0.000
	6.372	654.372	13.063	0.006	2.6666	1.8485	0.086	0.000	0.006	0.086			9.899	0.000
	6.464	654.464	13.059	0.006	2.6666	1.8485	0.096	0.000	0.006	0.096			9.895	0.000
	6.557	654.557	13.056	0.008	2.6666	1.8485	0.106	0.000	0.008	0.106			9.892	0.000
	6.651	654.651	13.053	0.006	2.6666	1.8485	0.116	0.000	0.006	0.116			9.889	0.000
	6.744	654.744	13.039	0.006	2.6666	1.8485	0.126	0.000	0.006	0.126			9.875	0.000
	6.854	654.854	13.035	0.006	2.6666	1.8485	0.138	0.000	0.006	0.138			9.871	0.000
	6.951	654.951	13.043	0.006	2.6666	1.8485	0.149	0.000	0.006	0.149			9.879	0.000
	7.046	655.046	13.049	0.006	2.6666	1.8485	0.159	0.000	0.006	0.159			9.885	0.000
	7.142	655.142	13.056	0.006	2.6666	1.8485	0.169	0.000	0.006	0.169			9.892	0.000
	7.234	655.234	13.034	0.006	2.6666	1.8485	0.179	0.000	0.006	0.179			9.870	0.000
	7.328	655.328	13.050	0.006	2.6666	1.8485	0.189	0.000	0.006	0.189			9.886	0.000
	7.423	655.423	13.062	0.006	2.6666	1.8485	0.200	0.000	0.006	0.200			9.898	0.000
	7.518	655.518	13.068	0.006	2.6666	1.8485	0.210	0.000	0.006	0.210			9.904	0.000
	7.611	655.611	13.084	0.006	2.6666	1.8485	0.220	0.000	0.006	0.220			9.920	0.000
	7.704	655.704	13.107	0.006	2.6666	1.8485	0.230	0.000	0.006	0.230			9.943	0.000
	7.800	655.800	13.111	0.006	2.6666	1.8485	0.241	0.000	0.006	0.241			9.947	0.000
	7.894	655.894	13.112	0.006	2.6666	1.8485	0.251	0.000	0.006	0.251			9.948	0.000
	7.988	655.988	13.142	0.008	2.6666	1.8485	0.261	0.000	0.008	0.261			9.978	0.000

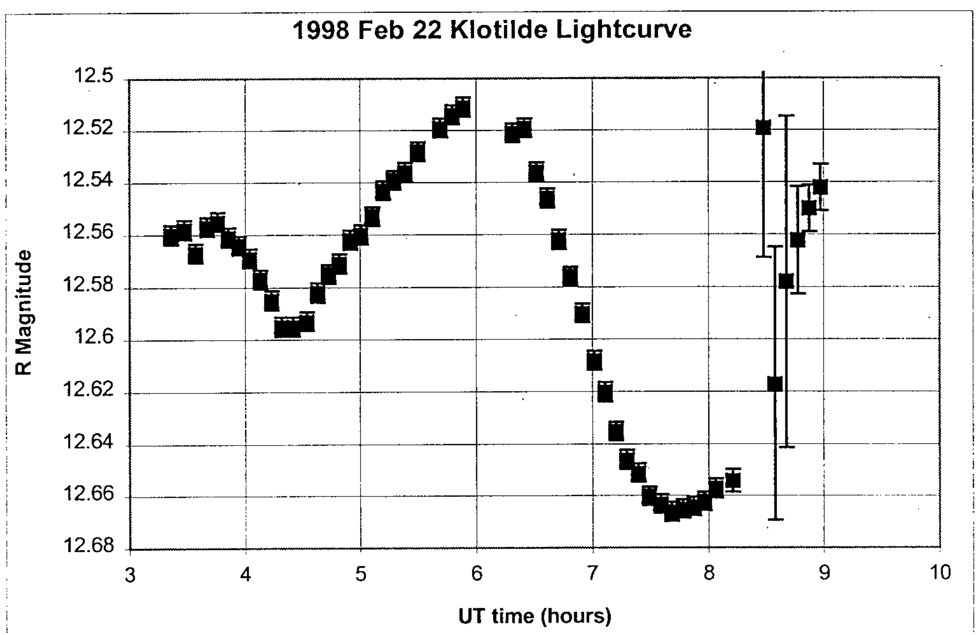
Combined Observations

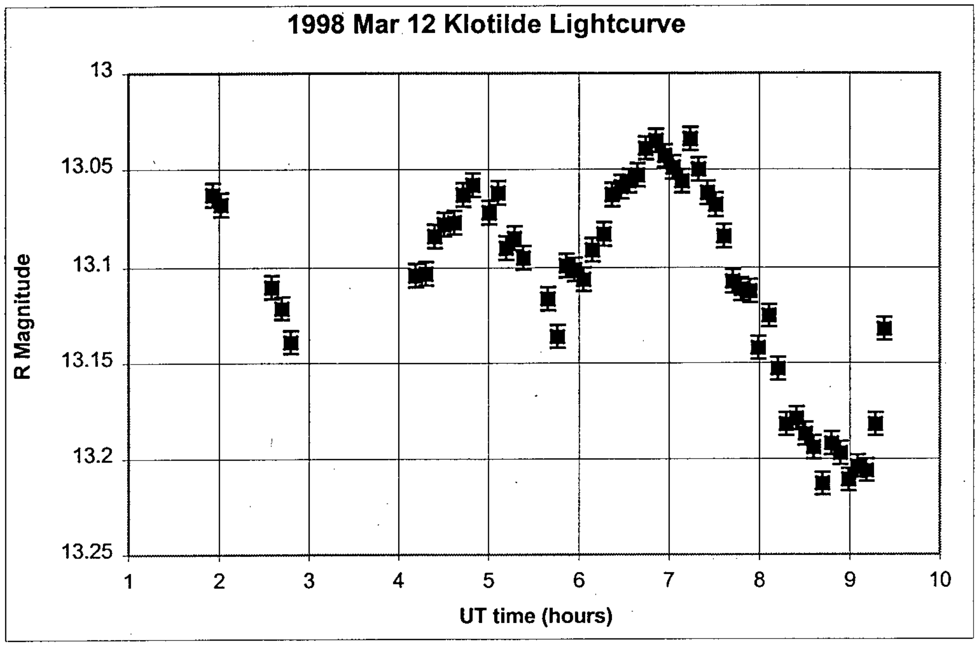
	8.109	656.109	13.125	0.009	2.6666	1.8485	0.274	0.000	0.009	0.274			9.961	0.000	
	8.203	656.203	13.153	0.011	2.6666	1.8485	0.284	0.000	0.011	0.284			9.989	0.000	
	8.299	656.299	13.182	0.007	2.6666	1.8485	0.295	0.000	0.007	0.295			10.018	0.000	
	8.412	656.412	13.179	0.007	2.6666	1.8485	0.307	0.000	0.007	0.307			10.015	0.000	
	8.506	656.506	13.187	0.008	2.6666	1.8485	0.317	0.000	0.008	0.317			10.023	0.000	
	8.600	656.600	13.194	0.008	2.6666	1.8485	0.328	0.000	0.008	0.328			10.030	0.000	
	8.695	656.695	13.213	0.009	2.6666	1.8485	0.338	0.000	0.009	0.338			10.049	0.000	
	8.796	656.796	13.192	0.009	2.6666	1.8485	0.349	0.000	0.009	0.349			10.028	0.000	
	8.893	656.893	13.197	0.009	2.6666	1.8485	0.359	0.000	0.009	0.359			10.033	0.000	
*	8.987	656.987	13.211	0.009	2.6666	1.8485	0.370	0.000	0.009	0.370			10.047	0.000	
	9.086	657.086	13.204	0.009	2.6666	1.8485	0.380	0.000	0.009	0.380			10.040	0.000	
	9.182	657.182	13.206	0.010	2.6666	1.8485	0.391	0.000	0.010	0.391			10.042	0.000	
	9.284	657.284	13.182	0.010	2.6666	1.8485	0.402	0.000	0.010	0.402			10.018	0.000	
	9.381	657.381	13.132	0.011	2.6666	1.8485	0.412	0.000	0.011	0.412			9.968	8.500	
13-Mar	2.830	674.830	13.030	0.005	2.6664	1.856	0.307	0.000	0.005	0.307				10.007	
	2.936	674.936	13.043	0.005	2.6664	1.856	0.318	0.000	0.005	0.318				10.020	
	3.032	675.032	13.049	0.005	2.6664	1.856	0.329	0.000	0.005	0.329				10.026	
	3.149	675.149	13.045	0.005	2.6664	1.856	0.341	0.000	0.005	0.341				10.022	
	3.249	675.249	13.058	0.005	2.6664	1.856	0.352	0.000	0.005	0.352				10.035	
	3.342	675.342	13.066	0.005	2.6664	1.856	0.362	0.000	0.005	0.362				10.043	
*	3.436	675.436	13.072	0.005	2.6664	1.856	0.372	0.000	0.005	0.372				10.049	
	3.533	675.533	13.065	0.005	2.6664	1.856	0.383	0.000	0.005	0.383				10.042	
	3.626	675.626	13.052	0.005	2.6664	1.856	0.393	0.000	0.005	0.393				10.029	
	3.723	675.723	13.049	0.005	2.6664	1.856	0.404	0.000	0.005	0.404				10.026	
	3.817	675.817	13.039	0.004	2.6664	1.856	0.414	0.000	0.004	0.414				10.016	
	3.911	675.911	13.025	0.004	2.6664	1.856	0.424	0.000	0.004	0.424				10.002	
	4.005	676.005	13.014	0.004	2.6664	1.856	0.434	0.000	0.004	0.434				9.991	
	4.099	676.099	12.992	0.004	2.6664	1.856	0.444	0.000	0.004	0.444				9.969	
	4.193	676.193	12.982	0.004	2.6664	1.856	0.454	0.000	0.004	0.454				9.959	
	4.286	676.286	12.970	0.004	2.6664	1.856	0.465	0.000	0.004	0.465				9.947	
	4.379	676.379	12.953	0.004	2.6664	1.856	0.475	0.000	0.004	0.475				9.930	
	4.473	676.473	12.959	0.004	2.6664	1.856	0.485	0.000	0.004	0.485				9.936	
	4.566	676.566	12.942	0.004	2.6664	1.856	0.495	0.000	0.004	0.495				9.919	
	4.659	676.659	12.939	0.004	2.6664	1.856	0.505	0.000	0.004	0.505				9.916	
	4.753	676.753	12.934	0.004	2.6664	1.856	0.515	0.000	0.004	0.515				9.911	
	4.847	676.847	12.931	0.004	2.6664	1.856	0.526	0.000	0.004	0.526				9.908	
	4.943	676.943	12.917	0.005	2.6664	1.856	0.536	0.000	0.005	0.536				9.894	

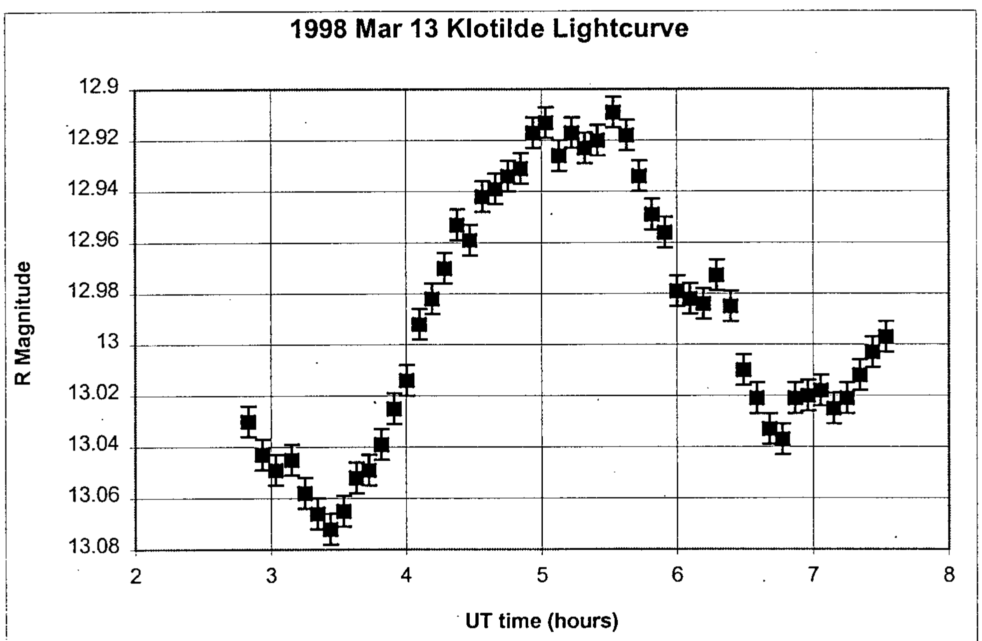
Combined Observations

	5.036	677.036	12.913	0.004	2.6664	1.856	0.546	0.000	0.004	0.546				9.890	
	5.1328	677.133	12.926	0.0045	2.6664	1.856	0.557	0.000	0.004	0.557				9.903	
	5.2264	677.226	12.917	0.0054	2.6664	1.856	0.567	0.000	0.005	0.567				9.894	
	5.3208	677.321	12.923	0.0054	2.6664	1.856	0.577	0.000	0.005	0.577				9.900	
	5.4147	677.415	12.92	0.0054	2.6664	1.856	0.587	0.000	0.005	0.587				9.897	
	5.5333	677.533	12.909	0.0054	2.6664	1.856	0.600	0.000	0.005	0.600				9.886	
	5.6297	677.630	12.918	0.0054	2.6664	1.856	0.611	0.000	0.005	0.611				9.895	
	5.7236	677.724	12.934	0.0054	2.6664	1.856	0.621	0.000	0.005	0.621				9.911	
	5.8167	677.817	12.949	0.0054	2.6664	1.856	0.631	0.000	0.005	0.631				9.926	
	5.9103	677.910	12.956	0.0054	2.6664	1.856	0.641	0.000	0.005	0.641				9.933	
	6.0033	678.003	12.979	0.0054	2.6664	1.856	0.651	0.000	0.005	0.651				9.956	
	6.0972	678.097	12.982	0.0054	2.6664	1.856	0.661	0.000	0.005	0.661				9.959	
	6.1919	678.192	12.984	0.0054	2.6664	1.856	0.672	0.000	0.005	0.672				9.961	
	6.2942	678.294	12.973	0.0054	2.6664	1.856	0.683	0.000	0.005	0.683				9.950	
	6.3978	678.398	12.985	0.0054	2.6664	1.856	0.694	0.000	0.005	0.694				9.962	
	6.4911	678.491	13.01	0.0054	2.6664	1.856	0.704	0.000	0.005	0.704				9.987	
	6.5867	678.587	13.021	0.0054	2.6664	1.856	0.714	0.000	0.005	0.714				9.998	
	6.6822	678.682	13.033	0.0054	2.6664	1.856	0.725	0.000	0.005	0.725				10.010	
	6.7764	678.776	13.037	0.0054	2.6664	1.856	0.735	0.000	0.005	0.735				10.014	
	6.8708	678.871	13.021	0.0054	2.6664	1.856	0.745	0.000	0.005	0.745				9.998	
	6.9639	678.964	13.02	0.0041	2.6664	1.856	0.755	0.000	0.004	0.755				9.997	
	7.0572	679.057	13.018	0.0041	2.6664	1.856	0.766	0.000	0.004	0.766				9.995	
	7.1517	679.152	13.025	0.0041	2.6664	1.856	0.776	0.000	0.004	0.776				10.002	
	7.2489	679.249	13.021	0.0041	2.6664	1.856	0.786	0.000	0.004	0.786				9.998	
	7.3428	679.343	13.012	0.0041	2.6664	1.856	0.797	0.000	0.004	0.797				9.989	
	7.4369	679.437	13.003	0.0041	2.6664	1.856	0.807	0.000	0.004	0.807				9.980	
	7.5333	679.533	12.997	0.0041	2.6664	1.856	0.817	0.000	0.004	0.817				9.974	


1998 Feb 13 Klotilde Lightcurve
R Magnitude
UT time (hours)
Food
Gas
Motel
Jan
Feb
Mar
Apr
May
Jun


1998 Feb 22 Klotilde Lightcurve
R Magnitude
UT time (hours)


1998 Mar 12 Klotilde Lightcurve
R Magnitude
13
13.05
13.1
13.15
13.2
13.25
1
2
3
4
5
6
7
8
9
10
UT time (hours)


1998 Mar 13 Klotilde Lightcurve
R Magnitude
12.9
12.92
12.94
12.96
12.98
13
13.02
13.04
13.06
13.08
2
3
4
5
6
7
8
UT time (hours)

APPENDIX B

Submitted Rotational Period Paper

CCD PHOTOMETRY OF 583 KLOTILDE AT THE U.S. AIR FORCE ACADEMY OBSERVATORY

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CCD photometry observations of asteroid 583 Klotilde, taken during 1998 February and March at the U.S. Air Force Academy Observatory are reported. The asteroid was observed as part of a masters degree thesis project. A synodic period of 9.210 ± 0.005 hours was determined from four nights of observations. The observed amplitude was 0.18 magnitudes.

All observations were made at the U. S. Air Force Academy Observatory. A Photometrics (PM512) CCD camera attached to a 61-cm Cassegrain telescope was used to take five minute exposures of the asteroid through a standard Johnson R-band filter. All images were processed using NOAO's IRAF package and differential photometry of the asteroid with respect to nearby stars was performed. Differential photometry measurements between several stars in each field were conducted to ensure the comparison stars were not variable. The data revealed that one of the comparison stars from one night was in fact variable. This star was not used as a comparison star. Since it is not identified as a variable star, further observations are currently being conducted to determine the nature of its variability.

Potential asteroid targets were selected by first setting an upper magnitude limit for asteroids at a favorable viewing position during February and March. Information provided by Harris (1997) was then consulted to see which of these asteroids did not have previously determined rotational periods. The asteroid 583 Klotilde was chosen as our primary target.

Asteroid 583 Klotilde was observed 48 times over a 5.5 hour period (4.8 to 10.3 hours UT) on UT 98 Feb 13; 52 times over a 5.6 hour period (3.4 to 9.0 hours UT) on UT 98 Feb 22; 56 times over a 7.5 hour period (1.9 to 9.4 hours UT) on UT 98 Mar 12; and 50 times over a 4.7 hour period (2.8 to 7.5 hours UT) on UT 98 Mar 13. The data from UT 98 Mar 12 suggests a triple minima lightcurve with each minimum having a distinctive depth. Combining this night with the next and visually inspecting the resulting lightcurve indicates a synodic period of 9.2 ± 0.1 hours containing three minima. Adding the observations from the other nights refines the synodic period to 9.210 ± 0.005 hours. The composite lightcurve using this period is shown in Figure 1. The observed amplitude was 0.18 magnitudes.

References:

Harris, A.W. (1997). "Minor Planet Lightcurve Parameters," Posted on WWW: http://cfa-www.harvard.edu/iau/lists/LightcurveDat.html (97 Oct 21 update).

Russell, J.L., Lasker, B.M., McLean, J.J., Sturch, C.R., & Jenker, H. (1990). "The Guide Star Catalog. II – Photometric and astrometric models and solutions." AJ 99, 2059-2081.

Figure Captions: (For figure submitted see Fig.4.5.)

Figure 1. Composite lightcurve for 583 Klotilde based on a 9.210 hour synodic period. Zero phase = UT 98 Mar 13, 00:00:00. Magnitudes calibrated roughly with the Hubble Guide Star Catalog magnitudes (Russell it al. 1990) of comparison stars, adjusted to common Earth and Sun distances (1 A.U. for both), and scaled to UT 98 Mar 13 data. Lightcurve is corrected for light travel time.

APPENDIX C

Computed Amata Coordinates / Input Data

Test case, Year, Month, Day.day, RA RA RA.ra, Dec Dec Dec.dec, magnitude, filter

1	1998	01	21.24164	03	44	50.43	42	12	41.6	15.9 V
	1998	01	21.24550	03	44	50.41	42	12	40.7	15.9 V
	1998	01	21.27009	03	44	50.55	42	12	32.5	16.0 V
	1998	01	21.27427	03	44	50.54	42	12	31.2	16.0 V
2	1998	01	22.21771	03	44	55.41	42	07	42.6	15.9 V
	1998	01	22.22047	03	44	55.42	42	07	41.2	15.9 V
	1998	01	22.27830	03	44	55.73	42	07	23.9	16.1 V
	1998	01	22.28038	03	44	55.74	42	07	23.1	16.1 V
3	1998	01	27.21590	03	45	46.10	41	43	02.5	16.5 V
	1998	01	27.21848	03	45	46.10	41	43	01.6	16.6 V
	1998	01	27.25006	03	45	46.54	41	42	53.6	15.7 V
	1998	01	27.25433	03	45	46.62	41	42	51.9	16.0 V
	1998	01	27.27655	03	45	46.94	41	42	45.1	16.0 V
4	1998	01	27.28177	03	45	47.03	41	42	43.5	16.0 V
5	1998	01	28.21384	03	46	01.17	41	38	18.7	16.1 V
	1998	01	28.21614	03	46	01.20	41	38	18.3	15.9 V
	1998	01	28.24293	03	46	01.63	41	38	10.2	16.0 V
	1998	01	28.24633	03	46	01.68	41	38	09.1	16.1 V
	1998	01	28.26623	03	46	01.96	41	38	03.8	16.0 V
6	1998	01	28.27069	03	46	02.02	41	38	02.5	16.0 V
1	1998	02	13.06861	03	53	21.41	40	32	49.2	16.6 V
2	1998	02	13.07160	03	53	21.53	40	32	48.7	16.5 V
3	1998	02	13.09028	03	53	22.22	40	32	44.8	16.4 V
4	1998	02	13.09264	03	53	22.33	40	32	44.0	16.4 V
5	1998	02	13.12376	03	53	23.49	40	32	37.6	16.4 V
6	1998	02	13.12638	03	53	23.59	40	32	37.1	16.4 V
1	1998	03	13.09222	04	18	34.54	39	20	22.9	16.2 V
2	1998	03	13.09472	04	18	34.71	39	20	22.4	16.4 V
3	1998	03	13.09684	04	18	34.84	39	20	22.2	16.1 V
4	1998	03	13.13267	04	18	37.23	39	20	18.7	16.2 V
5	1998	03	13.13863	04	18	37.63	39	20	17.6	16.3 V
6	1998	03	13.14065	04	18	37.76	39	20	17.3	16.1 V

APPENDIX D

Modified GEM Program

```
Program GEM (input, output);

{This program was originally produced by J.M.A. Danby.  Modifications
and alterations were made by 2Lt Dan Burtz in Mar 98 for thesis work for
a Master of Engineering degree in Space Operations from the University of
Colorado at Colorado Springs.  The data I used in this program was taken
from the U.S. Air Force Academy 16 inch telescope with the ST8 CCD camara.
Images taken were of asteroid 1035, Amata, from 21 Jan 98 through 13 Mar 98.

Equation numbers in parenthesis can be found in Fundamentals of Celestial
Mechanics by J.M.A. Danby, 2nd Ed., c1992.  Published by Willmann-Bell.

This program reads in data from an input file in the following format:
YYYY MM DD.DDDDD RA RA RA.RA DE DE DE.D
where RA and DE represent right ascension and declination in their
respective units.  The program also contains procedures which are commented
out that allow inputing these values in real time from the keyborad, and
for hardcoding them into the program.  The data input data is echo checked
to an output file along with the computed orbital elements.  Due to the
fact that the iterations in this program may not converge, continuing
with each iteration is specified by the user while running the program.
The final orbital elements are also printed to the screen.

(Notes from Danby)
Basic program for determining an orbit from three observations.
See section 7.3. The GEM method is used, where sector-triangle
ratios are found.  Allowance has NOT been made for planetary
aberration.  To turn this into a completely practical program,
this modification should be made, and possible more accurate
solar coordinates used.}

{-----------------------------------------------------------------
                     PROGRAM VARIABLES AND PROCEDURES

VARIABLES
  a          - Semi-major axis of orbit                             AU
  argp       - Argument of periapsis                                rad
  c1         - Scalar component of R vectors
  c3         - Scalar component of R vectors
  change     - Change in sum of rho's for each iteration            AU
  CONTINUE   - Loop control variable to continue with iterations    integer
  COUNT      - Iteration number                                     integer
  d          - Integer Julian day for input; Day of month for output integer
  dec        - Declination of observation                           rad
  decdeg     - Degree portion of input declination                  deg
  decmin     - Minute portion of input declination                  min
  decsec     - Second portion of input declination                  sec
  dt         -
  e          - Eccentricity
  f          -
  filein     - Variable designating the input data file
  fileout    - Variable designating the output data file
  fullday    - Julian day with fractional part
  g          -
  gd         - Geocentric distance, the magnitude of each rho       array of 3
  gk         - Gaussian constant  (sqrt of MU of sun)
  hc         - Heliocentric coordinates                             array
```

hour	- Right ascension hour portion	real
i	- Loop control variable	integer
inc	- Inclination of orbit	rad
j	- Loop control variable	integer
k	- Loop control variable	integer
kay	-	
m	- Month number in output	integer
MA	- Mean anomaly	rad
min	- Right ascension minute portion	min
n	- Mean motion	rad/day
omega	- Longitude of the ascending node through ecliptic	rad
p	- Semi-latus rectum	AU
pi	- Arc Cos(-1)	real
r	-	
r1	-	
r2	-	
ra	- Right ascension of observation	rad
rh	-	
rho1	-	
rho2	-	
rho3	-	
rm	-	
sec	- Right ascension seconds portion	sec
sc1	- Direction cosines of Solar coordinates	array
sc2	- Rotated solar coordinates	array
sx	-	
sy	-	
sz	-	
t	-	
tau1	- Time constant (delta t x gk)	
tau3	- Time constant (delta t x gk)	
temp	-	
time	- The Julian date of the observation time	days
tpc	-	
ut	- Decimal part of full input Julian Day	days
x	-	
x1	- Heliocentric equitorial coordinates for time 1	
x2	- Heliocentric equitorial coordinates for time 2	
xs	-	
xv	-	
y	-	
y1	- Sector triangle ratio for obs 1 - obs 2	unitless
y2	- Sector triangle ratio for obs 2 - obs 3	unitless
y3	- Sector triangle ratio for obs 1 - obs 3	unitless
ye	- Year of observation time; Year of calculated elements	integer
yg1	-	
yv	-	
z	-	
zv	-	

PROCEDURES

Coordinates_elements	- Calculates the orbital elements
Data	- Gets input times, RA, Dec; converts to working units
Date_Jd	- Converts a calendar date to a Julian date
Jd_Date	- Converts a Julian Date to a calendar date
Output	- Writes orbital elements to screen and file
Rotation	- Resolves vectors in Cunningham's reference system
Sector_triangle-ratio	- Uses Gauss' method to compute the ratio
Sun	- Calculates solor coordinates

FUNCTIONS

Atan2	- Finds the arc tangent of two numbers
Obliquity	- Calculates the obliquity of the ecliptic based on the JD
Q	- Calculates Q function for Gauss' method}

```
uses Crt, Dos;
Const
   pi: real = 3.1415926535897932;
   gk: real = 0.01720209895;

{-----------------------------------------------------------------------}

var
   fileout, filein:                               text;
   COUNT, CONTINUE:                               integer;
   ra, dec, time, sx, sy, sz:                     array[1..3] of real;
   rho1, rho2, rho3, gd:                          array[1..3] of real;
   rh, rm, sc1, sc2, hc:                          array[1..3, 1..3] of real;
   ye, m:                                         integer;
   d, ut, hour, min, sec, fullday:                real;
   decdeg, decmin, decsec:                        real;
   i, j, k:                                       integer;
   tau1, tau3, c1, c3:                            real;
   temp, change:                                  real;
   r1, r2, kay, dt, xs, yg1, y1, y2, y3:          real;
   x1, x2:                                        array[1..3] of real;
   r, x, y, z, xv, yv, zv, f, g:                  real;
   obl, a, e, inc, omega, argp, tpc, t:           real;
   n, p, MA:                                      real;

{-----------------------------------------------------------------------}

Procedure Date_Jd(var Year, Month: integer; Day, Ut: real;
                     var Jd: real);
var
   ryear: real;
Begin
   ryear := Year;
   Jd := 367*ryear - Int(1.75*(rYear + Int((Month + 9)/12))) + Day
         + Int(275*Month/9) + 1721013.5 + Ut/24;
end;

{-----------------------------------------------------------------------}

Procedure Jd_date(Jd: real; var Year, Month: integer; var Day: real);

{Finds the calendar date from the Julian day number. Uses the
procedure given by Jean Meeus in "Astronomical Formulas for
Calculators, pp 26, 27.}
var
   z, f, a, b, c, d: real;
   e: integer;

begin
   Jd := Jd + 0.5;
   z := int(Jd + 0.00001);
   f := Jd - z;
   if z < 2299161.0 then a := z
   else
      begin
         a := int((z - 1867216.25)/36524.25);
         a := z + 1 + a - int(a/4);
      end;
   b := a + 1524;
   c := int((b - 122.1)/365.25);
   d := int(365.25*c);
```

```
   e := trunc((b - d)/30.6001);
   Day := b - d - int(30.6001*e) + f;
   if  e < 13.5 then Month := e - 1 else Month := e - 13;
   if Month > 2.5 then Year := trunc(c) - 4716
   else Year := trunc(c) - 4715;
end;

{-----------------------------------------------------------------------}

function Obliquity (JD: real): real;

 { This function goes with the modified procedure Sun, both to be
   used with Danby's GEM program to calculate the heliocentric
   ecliptic orbital elements of a comet or asteroid referred to
   the mean ecliptic and equinox of J2000.0.

   Roger L. Mansfield, 1998 March 24. }

const
   RADIAN = 57.29577951;

begin
   Obliquity := 23.4392911/RADIAN;

end;

{-----------------------------------------------------------------------}

function Modf (X, Y: real): real;
begin
   Modf := Y*Frac(X/Y)
end;

{-----------------------------------------------------------------------}

Procedure SUN (JD: real; var X, Y, Z: real);

 { Given a Julian Ephemeris date JD, this procedure calculates the
   geocentric equatorial cartesian coordinates of the sun, in A.U.
   and referred to the mean equator and equinox of J2000.0.

   Test data for this procedure can be found in the Astronomical
   Almanac for the Year 1998, pp. C20-C23.

   Reference: Roger L. Mansfield, "Ephemeris of a Comet via
   Uniform Path Mechanics (UPM)," a Mathcad PLUS 6 worksheet
   at http://www.mathsoft.com (Math in Action files, Astronomy
   and Navigation Programs, September 1997); see procedural
   function HGEO. }

const
   RADIAN = 57.29577951;
   TPI = 6.283185307;
   SECPDG = 3600.0;
   OBLQTY = 23.4392911/RADIAN;
   SECPRV = 360.0*SECPDG;

var
   TC, A, ECC, K, N, ARGP, INC, LONG, M, S1M, S2M, V, CV, R,
   XPF, YPF, CP, SP, CI, SI, PX, PY, PZ, QX, QY, QZ, LM,
   XC, YC, ZC, CO, SO: real;


begin
```

```
{ Calculate Julian centuries elapsed from input JD to 2000
  January 1.5 TT. }

  TC := (JD - 2451545.0)/36525.0;

{ Calculate semimajor axis and eccentricity of Earth-moon
  barycenter's orbit around sun. }

  A := 1.00000011 - 0.00000005*TC;
  ECC := 0.01671022 - 0.00003804*TC;

{ Calculate mean motion of this orbit.  Note that A^(-3/2) =
  Exp(-1.5*Ln(A)). }

  K := 0.01720209895*Sqrt(1.00000304);
  N := K*Exp(-1.5*Ln(A));

{ Calculate argument of perigee, inclination, and longitude of
  perihelion. }

  ARGP := (102.94719 + 1198.28*TC/SECPDG)/RADIAN;
  INC := (0.00005 - 46.94*TC/SECPDG)/RADIAN;
  LONG := (100.46435 + (1293740.63 + 99.0*SECPRV)*TC/SECPDG)/RADIAN;

{ Calculate mean anomaly.  Calculate true anomaly using the equation
  of the center (Moulton, p. 171). }

  M := Modf(LONG-ARGP,TPI);
  S1M := Sin(M);
  S2M := Sin(2.0*M);
  V := M + ECC*(  2.0*S1M + ECC*(1.25*S2M +
          ECC*(13.0*Sin(3.0*M) - 3.0*S1M)*0.083333333 +
          ECC*(103.0*Sin(4.0*M) - 44.0*S2M)*0.010416667));

{ Calculate perifocal cartesian coordinates of Earth. }

  CV := Cos(V);
  R := A*(1.0 - ECC*ECC)/(1.0 + ECC*CV);
  XPF := R*CV;
  YPF := R*Sin(V);

{ Calculate ecliptic cartesian coordinates of Earth. }

  CP := Cos(ARGP);
  SP := Sin(ARGP);
  CI := Cos(INC);
  SI := Sin(INC);
  PX := CP;
  PY := CI*SP;
  PZ := SI*SP;
  QX :=  -SP;
  QY := CI*CP;
  QZ := SI*CP;
  X := XPF*PX + YPF*QX;
  Y := XPF*PY + YPF*QY;
  Z := XPF*PZ + YPF*QZ;

{ Correct from Earth-moon barycenter to geocenter using the
  mean orbital longitude of the moon.  Note that this correction
  is applied to the ecliptic cartesian coordinates. }

  LM := Modf(218.0 + 481268.0*TC, 360.0)/RADIAN;
  XC := X - 0.0000312*Cos(LM);
```

```
   YC := Y - 0.0000312*Sin(LM);
   ZC := Z;

 { Refer to the mean equator and equinox of J2000.0 by rotating
   about X-axis through angle OBLQTY, the obliquity of the
   ecliptic at J2000.0. }

   CO := Cos(OBLQTY);
   SO := Sin(OBLQTY);
   X := XC;
   Y := YC*CO - ZC*SO;
   Z := YC*SO + ZC*CO;

 { Change sign to convert heliocentric equatorial cartesian Earth
   vector to geocentric equatorial cartesian sun vector. }

   X := -X;
   Y := -Y;
   Z := -Z;

end;

{-----------------------------------------------------------------------}

Procedure Data;
begin
   Textmode(3);
   writeln(fileout, '                    Input Data Echo Check');
   for i := 1 to 3 do
      begin
      readln(filein, ye,m,fullday,hour,min,sec,decdeg,decmin,decsec);

      d := trunc(fullday);      ut := fullday - d;
      Date_Jd(ye, m, d, ut, time[i]);
      ra[i] := hour + (min + sec/60)/60;
      ra[i] := ra[i] *15*pi/180;
      dec[i] := decdeg + (decmin + decsec/60)/60;
      dec[i] := dec[i] *pi/180;

{Echo check input data.}
      writeln(fileout, 'Observation ',i);
      writeln(fileout, 'Year = ', ye, '   Mon = ', m,
                       '   Day = ', fullday);
      writeln(fileout, 'HA   = ', hour, 'h ',min, 'm ',sec, 's');
      writeln(fileout, 'DEC  = ', decdeg, 'deg ', decmin, 'm ',
                       decsec, 's');
      writeln(fileout);

      rh[i,1] := cos(ra[i]) * cos(dec[i]);
      rh[i,2] := sin(ra[i]) * cos(dec[i]);
      rh[i,3] := sin(dec[i]);

{Direction cosines.}
      Sun(time[i], sc1[i,1], sc1[i,2], sc1[i,3]);
      writeln;
   end;  {Observation loop.}

    tau1 := (time[1] - time[2]) * gk;
    tau3 := (time[3] - time[2]) * gk;
{(7.3.13).}
end;

{-----------------------------------------------------------------------}
```

```
{Procedure Data;
begin
   Textmode(3);
   for i := 1 to 3 do
      begin
         writeln('Enter data for observation ',i,':');
         writeln(' Time:');
         write('   year:     ');   readln(ye);
         write('   month:    ');   readln(m);
         write('   day:      ');   readln(d);
         write('   UT:       ');   readln(ut);   writeln;
         Date_Jd(ye, m, d, ut, time[i]);

         writeln(' Right ascension:');
         write('   hours:    ');   readln(hour);
         write('   minutes: ');   readln(min);
         write('   seconds: ');   readln(sec);   writeln;
         ra[i] := hour + (min + sec/60)/60;
         ra[i] := ra[i]*15*pi/180;
         writeln(' Declination (degrees minutes and seconds):');

         write('   degrees: ');   readln(decdeg);
         write('   minutes: ');   readln(degmin);
         write('   seconds: ');   readln(degsec);
         dec[i]  := decdeg + (decmin + decsec/60)/60;
         dec[i]  := dec[i]*pi/180;
         rh[i,1] := cos(ra[i])*cos(dec[i]);
         rh[i,2] := sin(ra[i])*cos(dec[i]);
         rh[i,3] := sin(dec[i]);}
{Direction cosines.}
        {Sun(time[i], scl[i,1], scl[i,2], scl[i,3]);
         writeln;
      end;}{Observation loop.}
  {tau1 := (time[1] - time[2])*gk;
   tau3 := (time[3] - time[2])*gk;}
{(7.3.13).}
{end;}

{-----------------------------------------------------------------------}

{Procedure Data;
begin
   Textmode(3);}
{Obs 1}
         {ye := 1998;   m := 1;   d := 21;   ut := 0.24164;
         Date_Jd(ye, m, d, ut, time[1]);

         hour  := 3;   min := 44;   sec := 50.43;
         ra[1] := hour + (min + sec/60)/60;
         ra[1] := ra[1]*15*pi/180;

         decdeg := 42;   decmin := 12;   decsec := 41.6;
         dec[1] := decdeg + (decmin + decsec/60)/60;
         dec[1] := dec[1]*pi/180;}
{Obs 2}
         {ye := 1998;   m := 2;   d := 13;   ut := 0.0716;
         Date_Jd(ye, m, d, ut, time[2]);

         hour  := 3;   min := 53;   sec := 21.41;
         ra[2] := hour + (min + sec/60)/60;
         ra[2] := ra[2]*15*pi/180;

         decdeg := 40;   decmin := 32;   decsec := 49.2;
         dec[2] := decdeg + (decmin + decsec/60)/60;
```

```
         dec[2] := dec[2]*pi/180;}
{Obs 3}
         {ye := 1998;   m := 3;   d := 13;   ut := 0.09222;
         Date_Jd(ye, m, d, ut, time[3]);

         hour  := 4;   min := 18;   sec := 34.54;
         ra[3] := hour + (min + sec/60)/60;
         ra[3] := ra[3]*15*pi/180;

         decdeg := 39;   decmin :=20;   decsec := 22.9;
         dec[3] := decdeg + (decmin + decsec/60)/60;
         dec[3] := dec[3]*pi/180;
   for i := 1 to 3 do
      begin
         rh[i,1] := cos(ra[i])*cos(dec[i]);
         rh[i,2] := sin(ra[i])*cos(dec[i]);
         rh[i,3] := sin(dec[i]);}
{Direction cosines.}
         {Sun(time[i], sc1[i,1], sc1[i,2], sc1[i,3]);
         writeln;
      end;} {Observation loop.}
    {tau1 := (time[1] - time[2])*gk;
    tau3 := (time[3] - time[2])*gk;}
{(7.3.13).}
{end;}

{--------------------------------------------------------------------}

Procedure Rotation;
{Resolves vectors in Cunningham's reference system.  See pp 228, 229.}
var z1, z2:  real;
begin
   z1 := 0;

   for i := 1 to 3 do
      begin
         rm[1,i] := rh[1,i];
         z1      := z1 + rh[1,i]*rh[3,i];
      end; {Loop for xi, unit vector of the first observation.}

   z2 := sqrt(1 - z1*z1);

   for i := 1 to 3 do
      rm[2,i] := (rh[3,i] - z1*rh[1,i])/z2;
{Loop for eta; see (7.3.10).}

   rm[3,1] := rm[1,2]*rm[2,3] - rm[1,3]*rm[2,2];
   rm[3,2] := rm[1,3]*rm[2,1] - rm[1,1]*rm[2,3];
   rm[3,3] := rm[1,1]*rm[2,2] - rm[1,2]*rm[2,1];
{Components of zeta; see (7.3.11).}
{Rotate the unit observation unit vectors.}

   for i := 1 to 3 do
      begin
         rho1[i] := 0;   rho2[i] := 0;   rho3[i] := 0;
      end;

   rho1[1] := 1;

{rho1[i] are lambda1, mu1, nu1: see (7.3.12).}
   for i := 1 to 3 do
      begin
         rho2[1] := rho2[1] + rm[1,i]*rh[2,i];
         rho2[2] := rho2[2] + rm[2,i]*rh[2,i];
```

```
          rho2[3] := rho2[3] + rm[3,i]*rh[2,i];
{lambda2, mu2, nu2.}
          rho3[1] := rho3[1] + rm[1,i]*rh[3,i];
          rho3[2] := rho3[2] + rm[2,i]*rh[3,i];
{lambda3 and mu3; nu3 = 0.}
      end;

{Now rotate the solar coordinates.}
   for i := 1 to 3 do
      begin
         for j := 1 to 3 do
            begin
               sc2[i,j] := 0;
               for k := 1 to 3 do
                  sc2[i,j] := sc2[i,j] + rm[i,k]*sc1[j,k];
             end;
      end;

{Note that the jth column of sc2 represents the solar coordinates
of the jth time of observation.}
end;

{-------------------------------------------------------------------}

Function Q: real;
{Calculates the function Q defined in (6.11.20). Uses formulas (6.11.29),
(6.11.31) and (6.11.32).}
var
   fac, y, y1, qt:      real;
   n:                   integer;
begin
   if abs(xs) < 0.1 then
      begin
{Use hypergeometric series (6.11.29).}
         fac := 1.2*xs;
         qt  := 1 + fac;

         for n := 1 to 10 do
            begin
               fac := fac*xs*(3 + n)/(2.5 + n);
               qt  := qt + fac;
            end; {n loop for hypergeometric series.}

         Q := qt;
      end

   else
      begin
         if xs > 0 then
            begin
               y  := 2*xs - 1;
               y1 := Sqrt(xs - xs*xs);
               Q  := (3/16)*(ArcTan(y/Sqrt(1 - y*y))
                      + 2*y*y1 + pi/2)/(y1*y1*y1);
{(6.11.31).}
            end

         else
            begin
               y := sqrt(xs*xs - xs);
               Q := (3/16)*(2*(1 - 2*xs)*y - ln(1 - 2*xs + 2*y))/(y*y*y);
{(6.11.32).}
            end;
      end;
```

```
end;

{--------------------------------------------------------------------}

Procedure Sector_triangle_ratio;
var ms, el, yg2, yg3, den, dy: real; nc: integer;

begin
   ms := (gk*dt)*(gk*dt)/(kay*kay*kay);
   el := (r1 + r2 - kay)/(2*kay);
{(6.12.11) and (6.12.12).}

   yg1 := 1;
{Initial guess.}
{Use Steffensen's method.}
   nc := 0;

   repeat
      nc  := nc + 1;
      xs  := ms/(yg1*yg1) - el;
      yg2 := 1 + (4*ms*Q)/(3*yg1*yg1);
{(6.12.14).}
      xs  := ms/(yg2*yg2) - el;
      yg3 := 1 + (4*ms*Q)/(3*yg2*yg2);
      den := (yg1 - 2*yg2 + yg3);
      if abs(den) > 1e-12 then
         begin
            dy  := (yg2 - yg1)*(yg2 - yg1)/(den);
            yg1 := yg1 - dy;
        end;

   until (abs(dy) < 1e-6) or (abs(den) < 1e-12) or (nc = 20);

   if nc = 20 then writeln('SECTOR-TRIANGLE ITERATION FAILED.');
end;

{--------------------------------------------------------------------}

Function Atan2(si, co: real): real;
{Finds the angle between 0 and 2*pi of which
the sine is proportional to si and the cosine is proportional to co.}
begin
   pi := 3.141592653589793;

   if si < 0 then
      begin
         if co = 0 then Atan2 := 1.5*pi
         else
            if co > 0 then Atan2 := 2*pi + ArcTan(si/co)
            else Atan2 := pi + ArcTan(si/co);
      end

   else
      begin
         if co = 0 then Atan2 := pi/2
                   else Atan2 := ArcTan(si/co);
         if co < 0 then Atan2 := pi + ArcTan(si/co);
      end;
end;

{--------------------------------------------------------------------}

Procedure Output;
begin
```

```
   writeln(' a (AU)               = ' ,a);
   writeln(' e (unitless)         = ' ,e);
   inc    := inc*180/pi;
   writeln(' Inclination (deg)    = ' ,inc);
   omega  := omega*180/pi;
   writeln(' Lon of A Node (deg) = ' ,omega);
   argp   := argp*180/pi;
   writeln(' Arg of Per (deg)     = ' ,argp);
   writeln(' n (deg/day)          = ' , n);
   writeln(' P (years)            = ' , p);
   Jd_Date(tpc, ye, m, d);
   writeln(' T, year              =  ' ,ye);
   writeln('    month             =  ' ,m);
   writeln('    day               =  ' ,d);

   writeln(fileout, '        Calculated Orbital Elements');
   writeln(fileout, ' a (AU)                 = ' ,a);
   writeln(fileout, ' e (unitless)           = ' ,e);
   writeln(fileout, ' Inclination (deg)      = ' ,inc);
   writeln(fileout, ' Long of A Node (deg) = ' ,omega);
   writeln(fileout, ' Arg of Per (deg)       = ' ,argp);
   writeln(fileout, ' n (deg/day)            = ' ,n);
   writeln(fileout, ' P (years)              = ', p);
   writeln(fileout, ' T, year                = ', ye);
   writeln(fileout, '    month               = ', m);
   writeln(fileout, '    day                 = ', d);
end;

{-------------------------------------------------------------------}

Procedure Coordinates_elements;
var
   mu, hx, hy, hz, h, ex, ey, ez, u, s, c, ean: real;
begin
   mu := gk*gk;
   hx := y*zv - z*yv;
   hy := z*xv - x*zv;
   hz := x*yv - y*xv;
   h  := Sqrt(hx*hx + hy*hy + hz*hz);
   r  := Sqrt(x*x + y*y + z*z);
   ex := (yv*hz - zv*hy)/mu - x/r;
   ey := (zv*hx - xv*hz)/mu - y/r;
   ez := (xv*hy - yv*hx)/mu - z/r;
   u  := x*xv + y*yv + z*zv;

   s  := hx;                    c := - hy;
   omega := Atan2(s,c);

   s  := Sqrt(hx*hx + hy*hy);   c := hz;
   inc := Atan2(s,c);

   s  := ez;                    c := (- ex*hy + ey*hx)/h;
   argp := Atan2(s,c);

   e  := Sqrt(ex*ex + ey*ey + ez*ez);
   a  := 1/(2/r - (xv*xv + yv*yv + zv*zv)/mu);
   n  := gk*(1/Sqrt(a*a*a));
   n  := n * 180.0/pi;
   p  := 360/n*(1/365.25);

   if a > 0 then
      begin
         s   := u/Sqrt(a*mu);     c := 1 - r/a;
         ean := Atan2(s,c);
```

```
            tpc := t - (ean - s)*Sqrt(a*a*a/mu);
         end

      else
         begin
            z   := (1 - r/a)/e;
            ean := ln(z + Sqrt(z*z - 1));
            if u < 0 then ean := - ean;
            tpc:= t - (e*(exp(ean) - exp(- ean))/2 - ean)*Sqrt(- a*a*a)/gk;
         end;

      Output;
   end;

   {-----------------------------------------------------------------}

   begin {Main program.}
      assign(filein, 'input.txt');       reset(filein);
      assign(fileout, 'output.txt');     rewrite(fileout);

      Data;
      Rotation;
   {Initialize.}
      c1 := tau3/(tau3 - tau1);
      c3 := - tau1/(tau3 - tau1);
   {(7.3.14).}

      for i := 1 to 3 do gd[i] := 0;
      writeln;
      COUNT := 0;

      repeat
         COUNT := COUNT + 1;
         writeln('c1 = ',c1);
         writeln('c3 = ',c3);

         temp   := (- c1*sc2[3,1] + sc2[3,2] - c3*sc2[3,3])/rho2[3];
         change := abs(temp - gd[2]);
         gd[2]  := abs(temp);
   {(7.3.15.}

         temp   := (gd[2]*rho2[2] + c1*sc2[2,1] - sc2[2,2] + c3*sc2[2,3])
                  /(c3*rho3[2]);
         change := change + abs(temp - gd[3]);
         gd[3]  := abs(temp);
   {(7.3.16).}

         temp   := (gd[2]*rho2[1] - c3*gd[3]*rho3[1]
                   + c1*sc2[1,1] - sc2[1,2] + c3*sc2[1,3])/(c1*rho1[1]);
         change := change + abs(temp - gd[1]);
         gd[1]  := abs(temp);
   {(7.3.17).}

         for i := 1 to 3 do writeln('rho',i,' = ',gd[i]);

         if (change > 0.001) then
            begin
   {Find heliocentric coordinates.}
               for i := 1 to 3 do
                  begin
                     hc[i,1] := gd[1]*rho1[i] - sc2[i,1];
                     hc[i,2] := gd[2]*rho2[i] - sc2[i,2];
                     hc[i,3] := gd[3]*rho3[i] - sc2[i,3];
                  end;
```

```
{The jth column of hc represents the jth observation.}

{Now find the sector-triangle ratios.}
  {Find Y1}
            r1 := 0;   r2 := 0;   kay := 0;

            for i := 1 to 3 do
               begin
                  r1  := r1  + hc[i,2]*hc[i,2];
                  r2  := r2  + hc[i,3]*hc[i,3];
                  kay := kay + hc[i,2]*hc[i,3];
               end;

            r1  := sqrt(r1);
            r2  := sqrt(r2);
            kay := sqrt(2*kay + 2*r1*r2);
            dt  := time[3] - time[2];
            Sector_triangle_ratio;
            y1  := yg1;
            writeln('Y1 = ', y1);

  {Find Y2}
            r1  := 0;      r2  := 0;       kay := 0;

            for i := 1 to 3 do
               begin
                  r1  := r1  + hc[i,1]*hc[i,1];
                  r2  := r2  + hc[i,3]*hc[i,3];
                  kay := kay + hc[i,1]*hc[i,3];
               end;

            r1  := sqrt(r1);
            r2  := sqrt(r2);
            kay := sqrt(2*kay + 2*r1*r2);
            dt  := time[3] - time[1];
            Sector_triangle_ratio;
            y2  := yg1;
            writeln('Y2 = ', y2);

  {Find Y3}
            r1  := 0;       r2  := 0;        kay := 0;

            for i := 1 to 3 do
               begin
                  r1  := r1  + hc[i,1]*hc[i,1];
                  r2  := r2  + hc[i,2]*hc[i,2];
                  kay := kay + hc[i,1]*hc[i,2];
               end;

            r1  := sqrt(r1);
            r2  := sqrt(r2);
            kay := sqrt(2*kay + 2*r1*r2);
            dt  := time[2] - time[1];
            Sector_triangle_ratio;
            y3  := yg1;
            writeln('Y3 = ',y3);

 {Update guess for c1 and c3}
            c1 := (y2/y1)*(tau3)/(tau3 - tau1);
            c3 := (y2/y3)*( - tau1)/(tau3 - tau1);

         end; {Loop for iterating on c1 and c3.}

         writeln('change = ',change);
```

```
        writeln;
        writeln('COUNT = ',COUNT:3);
        write  ('CONTINUE (1) OR STOP (0) ? ');
        readln (CONTINUE);
        writeln;
   until (change < 0.001) or (keypressed) or (COUNT = 100) or (CONTINUE = 0);
{The 'small quantity' can be changed at your discretion.}

{Find heliocentric equatorial coordinates for times 1 and 2.}

   for i := 1 to 3 do
      begin
         x1[i] := gd[1]*rh[1,i] - sc1[1,i];
         x2[i] := gd[2]*rh[2,i] - sc1[2,i];
      end;

   r := sqrt(x1[1]*x1[1] + x1[2]*x1[2] + x1[3]*x1[3]);
   f := 1 - 2*gk*gk*dt*dt/(kay*kay*yg1*yg1*r);
   g := dt/yg1;
{Find velocity components at time 1.}

   for i := 1 to 3 do
      x2[i] := (x2[i] - f*x1[i])/g;

{Rotate to ecliptic coordinates.}
   obl := Obliquity(time[2]);
   x   := x1[1];
   y   := x1[2]*cos(obl) + x1[3]*sin(obl);
   z   := - x1[2]*sin(obl) + x1[3]*cos(obl);
   xv  := x2[1];
   yv  := x2[2]*cos(obl) + x2[3]*sin(obl);
   zv  := - x2[2]*sin(obl) + x2[3]*cos(obl);

{Find elements.}
   t := time[1];
   Coordinates_elements;

   close(filein);
   close(fileout);
end.
```

APPENDIX E

GEM Output Data

TEST CASE 1

Input Data Echo Check

Observation 1
Year = 1998   Mon = 1   Day = 2.1241640000E+01
HA = 3.0000000000E+00h 4.4000000000E+01m 5.0430000000E+01s
DEC = 4.2000000000E+01deg 1.2000000000E+01m 4.1600000000E+01s

Observation 2
Year = 1998   Mon = 2   Day = 1.3068610000E+01
HA = 3.0000000000E+00h 5.3000000000E+01m 2.1410000000E+01s
DEC = 4.0000000000E+01deg 3.2000000000E+01m 4.9200000000E+01s

Observation 3
Year = 1998   Mon = 3   Day = 1.3092220000E+01
HA = 4.0000000000E+00h 1.8000000000E+01m 3.4540000000E+01s
DEC = 3.9000000000E+01deg 2.0000000000E+01m 2.2900000000E+01s

Calculated Orbital Elements

a (AU)	= 3.2182660546E+00
e (unitless)	= 1.5972662894E-01
Inclination (deg)	= 1.8062012065E+01
Long of A Node (deg)	= 1.4028081831E+00
Arg of Per (deg)	= 3.2927072274E+02
n (deg/day)	= 1.7071479356E-01
P (years)	= 5.7735259071E+00
T, year	= 1996
month	= 8
day	= 6.8729515076E+00

TEST CASE 2

Input Data Echo Check

Observation 1
Year = 1998 Mon = 1 Day = 2.2217710000E+01
HA = 3.0000000000E+00h 4.4000000000E+01m 5.5410000000E+01s
DEC = 4.2000000000E+01deg 7.0000000000E+00m 4.2600000000E+01s

Observation 2
Year = 1998 Mon = 2 Day = 1.3071600000E+01
HA = 3.0000000000E+00h 5.3000000000E+01m 2.1530000000E+01s
DEC = 4.0000000000E+01deg 3.2000000000E+01m 4.8700000000E+01s

Observation 3
Year = 1998 Mon = 3 Day = 1.3094720000E+01
HA = 4.0000000000E+00h 1.8000000000E+01m 3.4710000000E+01s
DEC = 3.9000000000E+01deg 2.0000000000E+01m 2.2400000000E+01s

Calculated Orbital Elements

a (AU)	= 3.2101537365E+00
e (unitless)	= 1.6329192875E-01
Inclination (deg)	= 1.8063142247E+01
Long of A Node (deg)	= 1.4833787017E+00
Arg of Per (deg)	= 3.2862022751E+02
n (deg/day)	= 1.7136231735E-01
P (years)	= 5.7517095859E+00
T, year	= 1996
month	= 8
day	= 7.6320877075E+00

TEST CASE 3

Input Data Echo Check

Observation 1
Year = 1998 Mon = 1 Day = 2.7215900000E+01
HA = 3.0000000000E+00h 4.5000000000E+01m 4.6100000000E+01s
DEC = 4.1000000000E+01deg 4.3000000000E+01m 2.5000000000E+00s

Observation 2
Year = 1998 Mon = 2 Day = 1.3090280000E+01
HA = 3.0000000000E+00h 5.3000000000E+01m 2.2220000000E+01s
DEC = 4.0000000000E+01deg 3.2000000000E+01m 4.4800000000E+01s

Observation 3
Year = 1998 Mon = 3 Day = 1.3096840000E+01
HA = 4.0000000000E+00h 1.8000000000E+01m 3.4840000000E+01s
DEC = 3.9000000000E+01deg 2.0000000000E+01m 2.2200000000E+01s

Calculated Orbital Elements

a (AU)	= 3.2268883584E+00
e (unitless)	= 1.5592254265E-01
Inclination (deg)	= 1.8061130253E+01
Long of A Node (deg)	= 1.3648144047E+00
Arg of Per (deg)	= 3.2991056136E+02
n (deg/day)	= 1.7003102133E-01
P (years)	= 5.7967438862E+00
T, year	= 1996
month	= 8
day	= 5.9996833801E+00

TEST CASE 4

Input Data Echo Check

Observation 1
Year = 1998 Mon = 1 Day = 2.7281770000E+01
HA = 3.0000000000E+00h 4.5000000000E+01m 4.7030000000E+01s
DEC = 4.1000000000E+01deg 4.2000000000E+01m 4.3500000000E+01s

Observation 2
Year = 1998 Mon = 2 Day = 1.3092640000E+01
HA = 3.0000000000E+00h 5.3000000000E+01m 2.2330000000E+01s
DEC = 4.0000000000E+01deg 3.2000000000E+01m 4.4000000000E+01s

Observation 3
Year = 1998 Mon = 3 Day = 1.3132670000E+01
HA = 4.0000000000E+00h 1.8000000000E+01m 3.7230000000E+01s
DEC = 3.9000000000E+01deg 2.0000000000E+01m 1.8700000000E+01s

Calculated Orbital Elements

a (AU)	= 3.2727041601E+00
e (unitless)	= 1.2374729827E-01
Inclination (deg)	= 1.8049346092E+01
Long of A Node (deg)	= 9.5408620780E-01
Arg of Per (deg)	= 3.3445223269E+02
n (deg/day)	= 1.6647305562E-01
P (years)	= 5.9206355028E+00
T, year	= 1996
month	= 7
day	= 2.9782054901E+01

TEST CASE 5

Input Data Echo Check

Observation 1
Year = 1998   Mon = 1   Day = 2.8213840000E+01
HA = 3.0000000000E+00h 4.6000000000E+01m 1.1700000000E+00s
DEC = 4.1000000000E+01deg 3.8000000000E+01m 1.8700000000E+01s

Observation 2
Year = 1998   Mon = 2   Day = 1.3123760000E+01
HA = 3.0000000000E+00h 5.3000000000E+01m 2.3490000000E+01s
DEC = 4.0000000000E+01deg 3.2000000000E+01m 3.7600000000E+01s

Observation 3
Year = 1998   Mon = 3   Day = 1.3138630000E+01
HA = 4.0000000000E+00h 1.8000000000E+01m 3.7630000000E+01s
DEC = 3.9000000000E+01deg 2.0000000000E+01m 1.7600000000E+01s

Calculated Orbital Elements

a (AU)	= 3.2071182132E+00
e (unitless)	= 1.6607123946E-01
Inclination (deg)	= 1.8061309603E+01
Long of A Node (deg)	= 1.5977124414E+00
Arg of Per (deg)	= 3.2851102974E+02
n (deg/day)	= 1.7160566543E-01
P (years)	= 5.7435532848E+00
T, year	= 1996
month	= 8
day	= 1.0176685333E+01

TEST CASE 6

Input Data Echo Check

Observation 1
Year = 1998 Mon = 1 Day = 2.8270690000E+01
HA = 3.0000000000E+00h 4.6000000000E+01m 2.0200000000E+00s
DEC = 4.1000000000E+01deg 3.8000000000E+01m 2.5000000000E+00s

Observation 2
Year = 1998 Mon = 2 Day = 1.3126380000E+01
HA = 3.0000000000E+00h 5.3000000000E+01m 2.3590000000E+01s
DEC = 4.0000000000E+01deg 3.2000000000E+01m 3.7100000000E+01s

Observation 3
Year = 1998 Mon = 3 Day = 1.3140650000E+01
HA = 4.0000000000E+00h 1.8000000000E+01m 3.7760000000E+01s
DEC = 3.9000000000E+01deg 2.0000000000E+01m 1.7300000000E+01s

Calculated Orbital Elements

a (AU)	= 3.2503598737E+00
e (unitless)	= 1.4314015144E-01
Inclination (deg)	= 1.8053590108E+01
Long of A Node (deg)	= 1.1920248958E+00
Arg of Per (deg)	= 3.3218643621E+02
n (deg/day)	= 1.6819260693E-01
P (years)	= 5.8601046821E+00
T, year	= 1996
month	= 8
day	= 4.7938117981E+00

APPENDIX F

Modified ORBDET Program

```
(*-----------------------------------------------------------------------*)
(*                               ORBDET                                  *)
(*        Gaussian orbit determination from three observations          *)
(*              using the abbreviated method of Bucerius                 *)
(*                         version 93/07/01                              *)
(*-----------------------------------------------------------------------*)

PROGRAM ORBDET(INPUT,OUTPUT,ORBINP,ORBOUT);

  USES MATLIB, PNULIB, SPHLIB, SUNLIB, TIMLIB, KEPLIB;

  TYPE CHAR80 = ARRAY[1..80] OF CHAR;

  VAR  TEQX                              : REAL;
       TP,Q,ECC,INC,LAN,AOP       : REAL;
       JDO                               : REAL3;
       RSUN,E                            : MAT3X;
       HEADER                            : CHAR80;
       ORBINP,ORBOUT              : TEXT;
       MA                                : REAL;


(*-----------------------------------------------------------------------*)
(* START: reads the input file and preprocesses the observational data   *)
(*                                                                       *)
(* output:                                                               *)
(*   RSUN:  matrix of three Sun position vectors in ecliptic coordinates *)
(*   E:     matrix of three observation direction unit vectors           *)
(*   JD:    julian date of the three observation times                  *)
(*   TEQX:  equinox of RSUN and E (in Julian centuries since J2000)      *)
(*-----------------------------------------------------------------------*)

PROCEDURE START (VAR HEADER: CHAR80;
                 VAR RSUN,E: MAT3X; VAR JDO: REAL3; VAR TEQX: REAL);

  VAR DAY,MONTH,YEAR,D,M,I :  INTEGER;
      UT,S,DUMMY                      :  REAL;
      EQXO,EQX,TEQXO                  :  REAL;
      LS,BS,RS,LP,BP,RA,DEC,T         :  REAL3;
      A,AS                                    :  REAL33;
      ORBINP                                  :  TEXT;

  BEGIN

    (* open input file *)

    (* RESET(ORBINP); *)                             (* Standard Pascal *)
    ASSIGN(ORBINP,'ORBINP.DAT'); RESET(ORBINP);      (* Turbo Pascal *)

    (* read data from file ORBINP *)

    FOR I:=1 TO 80 DO                                (* header *)
      IF NOT(EOLN(ORBINP)) THEN READ(ORBINP,HEADER[I]) ELSE HEADER[I]:=' ';
    READLN(ORBINP);
    FOR I := 1 TO 3 DO                               (* 3 observations *)
```

```
  BEGIN
    READ   (ORBINP,YEAR,MONTH,DAY,UT);                          (* date *)
    READ   (ORBINP,D,M,S); DDD(D,M,S,RA[I]);                    (* RA   *)
    READLN(ORBINP,D,M,S); DDD(D,M,S,DEC[I]);                    (* Dec  *)
    RA[I]:=RA[I]*15.0;
    JDO[I] := 2400000.5+MJD(DAY,MONTH,YEAR,UT);
    T[I]   := (JDO[I]-2451545.0)/36525.0;
  END;
WRITELN;
READLN(ORBINP,EQXO); TEQXO:=(EQXO-2000.0)/100.0;  (* equinox  *)

(* desired equinox of the orbital elements *)

READ(ORBINP,EQX ); TEQX :=(EQX -2000.0)/100.0;

(* calculate initial data of the orbit determination *)

PMATECL(TEQXO,TEQX,A);
FOR I := 1 TO 3 DO
  BEGIN
    CART    (1.0,DEC[I],RA[I],E[I,X],E[I,Y],E[I,Z]);
    EQUECL (      TEQXO       ,E[I,X],E[I,Y],E[I,Z]);
    PRECART(       A          ,E[I,X],E[I,Y],E[I,Z]);
    POLAR   (E[I,X],E[I,Y],E[I,Z],DUMMY,BP[I],LP[I]);
    PMATECL( T[I],TEQX,AS);
    SUN200 (T[I],LS[I],BS[I],RS[I]);
    CART    (RS[I],BS[I],LS[I],RSUN[I,X],RSUN[I,Y],RSUN[I,Z]);
    PRECART(       AS           ,RSUN[I,X],RSUN[I,Y],RSUN[I,Z]);
  END;

(* open file for writing *)

(* REWRITE(ORBOUT); *)                              (* Standard Pascal *)
ASSIGN(ORBOUT,'ORBOUT.DAT'); REWRITE(ORBOUT);       (* Turbo Pascal *)

WRITELN('   ORBDET: orbit determination from three observations       ');
WRITELN('                        version 93/07/01                        ');
WRITELN('         (c) 1993 Thomas Pfleger, Oliver Montenbruck         ');
WRITELN; WRITELN;
WRITELN(' Summary of orbit determination ');
WRITELN;
WRITE  ('  '); FOR I:=1 TO 78 DO WRITE(HEADER[I]); WRITELN;
WRITELN;
WRITELN(' Initial data (ecliptic geocentric coordinates (in deg))');
WRITELN;
WRITELN('  Julian Date         ', JDO[1]:12:2,JDO[2]:12:2,JDO[3]:12:2);
WRITELN('  Solar longitude     ',  LS[1]:12:2, LS[2]:12:2, LS[3]:12:2);
WRITELN('  Planet/Comet Longitude',LP[1]:9:2, LP[2]:12:2, LP[3]:12:2);
WRITELN('  Planet/Comet Latitude ',BP[1]:9:2, BP[2]:12:2, BP[3]:12:2);
WRITELN; WRITELN;

WRITELN(ORBOUT,'   ORBDET: orbit determination from three observations ');
WRITELN(ORBOUT,'                        version 93/07/01                  ');
WRITELN(ORBOUT,'         (c) 1993 Thomas Pfleger, Oliver Montenbruck     ');
WRITELN(ORBOUT); WRITELN(ORBOUT);
WRITELN(ORBOUT,' Summary of orbit determination ');
WRITELN(ORBOUT);
WRITE  (ORBOUT,'  '); FOR I:=1 TO 78 DO WRITE(HEADER[I]); WRITELN;
WRITELN(ORBOUT);
WRITELN(ORBOUT,' Initial data (ecliptic geocentric coordinates (in deg))');
WRITELN(ORBOUT);
WRITELN(ORBOUT,'         Julian     Date                                  ',
JDO[1]:12:2,JDO[2]:12:2,JDO[3]:12:2);
```

```
    WRITELN(ORBOUT,'     Solar   longitude          ',     LS[1]:12:2,   LS[2]:12:2,
LS[3]:12:2);
    WRITELN(ORBOUT,'        Planet/Comet    Longitude',LP[1]:9:2,     LP[2]:12:2,
LP[3]:12:2);
    WRITELN(ORBOUT,'       Planet/Comet    Latitude    ',BP[1]:9:2,    BP[2]:12:2,
BP[3]:12:2);
    WRITELN(ORBOUT); WRITELN(ORBOUT);

  END;

(*-------------------------------------------------------------------- *)
(* DUMPELEM: output of orbital elements (screen)                         *)
(*-------------------------------------------------------------------- *)
PROCEDURE DUMPELEM(TP,Q,ECC,INC,LAN,AOP,TEQX:REAL);
  CONST KGAUSS = 0.01720209895;     PI = 3.141592654;
  VAR DAY,MONTH,YEAR        : INTEGER;
      MODJD,UT                     : REAL;
      AX,MM,MA,PER                 : REAL;
  BEGIN

    MODJD := TP*36525.0 + 51544.5;
    CALDAT( MODJD, DAY,MONTH,YEAR,UT);

    AX  := Q/(1.0-ECC);
    MM  := KGAUSS / SQRT(ABS(AX*AX*AX));
    MA  := MM * (2450800.5-(MODJD+2400000.5));
    MM  := MM * 180.0/PI;
    MA  := MA * 180.0/PI;
    PER := 360.0/MM * (1/365.25);

    WRITELN(' Orbital elements',
            ' (Equinox ','J',100.0*TEQX+2000.0:8:2,')');
    WRITELN;
    WRITELN('  Perihelion date       tp      ',
            YEAR:4,'/',MONTH:2,'/',DAY:2,UT:8:4,'h',
            '   (JD',MODJD+2400000.5:11:2,')');
    WRITELN('  Perihelion distance  q[AU] ',  Q:12:6);
    WRITELN('  Semi-major axis      a[AU] ',  Q/(1-ECC):12:6);
    WRITELN('  Eccentricity         e     ',  ECC:12:6);
    WRITELN('  Inclination          i     ',  INC:10:4,' degrees');
    WRITELN('  Ascending node       Omega ',  LAN:10:4,' degrees');
    WRITELN('  Long. of perihelion  pi    ',  AOP+LAN:10:4,' degrees');
    WRITELN('  Arg. of perihelion   omega ',  AOP:10:4,' degrees');
    WRITELN('  Mean motion          n     ',  MM:12:6,' degrees/day');
    WRITELN('  Mean anomaly         M     ',  MA:12:6,' degrees');
    WRITELN('  Orbital period       P     ',  PER:10:4,' years');
    WRITELN;
  END;

(*--------------------------------------------------------------------*)
(* SAVEELEM: output of orbital elements (file)                          *)
(*--------------------------------------------------------------------*)
PROCEDURE SAVEELEM(TP,Q,ECC,INC,LAN,AOP,TEQX:REAL;
                   HEADER: CHAR80);

  CONST KGAUSS = 0.01720209895;      PI = 3.141592654;
  VAR I,DAY,MONTH,YEAR      : INTEGER;
      MODJD,UT                     : REAL;
      AX,MM,MA,PER                 : REAL;

  BEGIN

    MODJD := TP*36525.0 + 51544.5;
    CALDAT( MODJD, DAY,MONTH,YEAR,UT);
```

```
    AX  := Q/(1.0-ECC);
    MM  := KGAUSS / SQRT(ABS(AX*AX*AX));
    MA  := MM * (2450800.5-(MODJD+2400000.5));
    MM  := MM * 180.0/PI;
    MA  := MA * 180.0/PI;
    PER := 360.0/MM * (1/365.25);

    WRITE  (ORBOUT,YEAR:5,MONTH:3,(DAY+UT/24.0):7:3,'!':6);
    WRITELN(ORBOUT,' perihelion time T0 (y m d.d)  =  JD ',
                   (MODJD+2400000.5):12:3);
    WRITELN(ORBOUT, Q :12:6,'!': 9,' q  ( a =',Q/(1-ECC):10:6,' )');
    WRITELN(ORBOUT,ECC:12:6,'!': 9,' e ');
    WRITELN(ORBOUT,INC:10:4,'!':11,' i ');
    WRITELN(ORBOUT,LAN:10:4,'!':11,' long.asc.node ');
    WRITELN(ORBOUT,AOP:10:4,'!':11,
                   ' arg.perih. ( long.per. = ',AOP+LAN:9:4,' )');
    WRITELN(ORBOUT,MM:12:6,'!': 9,' n ');
    WRITELN(ORBOUT,MA:12:6,'!': 9,' M ');
    WRITELN(ORBOUT,PER:10:4,'!':11,' P ');
    WRITELN(ORBOUT,TEQX*100.0+2000.0:8:2,'!':13,' equinox (J)');
    WRITE  (ORBOUT,'! ');
    FOR I:=1 TO 78 DO WRITE(ORBOUT,HEADER[I]);

    RESET(ORBOUT); (* close file *)

  END;

(*-----------------------------------------------------------------------*)
(* RETARD: light-time correction                                         *)
(*   JDO: times of observation (t1',t2',t3') (Julian Date)               *)
(*   RHO: three geocentric distances (in AU)                             *)
(*   JD:  times of light emittance (t1,t2,t3) (Julian Date)              *)
(*   TAU: scaled time differences                                        *)
(*-----------------------------------------------------------------------*)
PROCEDURE RETARD ( JDO,RHO: REAL3; VAR JD,TAU: REAL3);
  CONST KGAUSS = 0.01720209895;  A = 0.00578;
  VAR   I: INTEGER;
  BEGIN
    FOR I:=1 TO 3 DO  JD[I]:=JDO[I]-A*RHO[I];
    TAU[1] := KGAUSS*(JD[3]-JD[2]);  TAU[2] := KGAUSS*(JD[3]-JD[1]);
    TAU[3] := KGAUSS*(JD[2]-JD[1]);
  END;

(*-----------------------------------------------------------------------*)
(* GAUSS: iteration of the abbreviated Gauss method                      *)
(*                                                                       *)
(*  RSUN: three vectors of geocentric Sun positions                      *)
(*  E   : three unit vectors of geocentric observation directions        *)
(*  JDO : three observation times (Julian Date)                          *)
(*  TP  : time of perihelion passage (Julian centuries since J2000)      *)
(*  Q   : perihelion distance                                            *)
(*  ECC : eccentricity                                                   *)
(*  INC : inclination                                                    *)
(*  LAN : longitude of the ascending node                                *)
(*  AOP : argument of perihelion                                         *)
(*-----------------------------------------------------------------------*)

PROCEDURE GAUSS ( RSUN,E: MAT3X; JDO:REAL3;
                  VAR TP,Q,ECC,INC,LAN,AOP: REAL );

  CONST EPS_RHO =1.0E-8;

  VAR I,J                   : INTEGER;
```

```
    S                         : INDEX;
    RHOOLD,DET          : REAL;
    JD,RHO,N,TAU,ETA    : REAL3;
    DI                        : VECTOR;
    RPL                       : MAT3X;
    DD                        : REAL33;

BEGIN

  (* calculate initial approximations of n1 and n3 *)

  N[1] := (JDO[3]-JDO[2]) / (JDO[3]-JDO[1]);     N[2] := -1.0;
  N[3] := (JDO[2]-JDO[1]) / (JDO[3]-JDO[1]);

  (* calculate matrix D and its determinant (det(D) = e3.d3) *)

  CROSS(E[2],E[3],DI);  FOR J:=1 TO 3 DO DD[1,J]:=DOT(DI,RSUN[J]);
  CROSS(E[3],E[1],DI);  FOR J:=1 TO 3 DO DD[2,J]:=DOT(DI,RSUN[J]);
  CROSS(E[1],E[2],DI);  FOR J:=1 TO 3 DO DD[3,J]:=DOT(DI,RSUN[J]);
  DET := DOT(E[3],DI);

  WRITELN; WRITELN(' Iteration of the geocentric distances rho [AU] ');
  WRITELN;

  RHO[2] := 0;

  (* Iterate until distance rho[2] does not change any more *)

  RHO[2] := 0;

  REPEAT

     RHOOLD := RHO[2];

    (* geocentric distance rho from n1 and n3 *)
    FOR I := 1 TO 3 DO
      RHO[I]:=( N[1]*DD[I,1] - DD[I,2] + N[3]*DD[I,3] ) / (N[I]*DET);

    (* apply light-time correction and calculate time differences *)
    RETARD (JDO,RHO,JD,TAU);

    (* heliocentric coordinate vectors *)
    FOR I := 1 TO 3 DO
      FOR S := X TO Z DO
        RPL[I,S] := RHO[I]*E[I,S]-RSUN[I,S];

    (* sector/triangle ratios eta[i] *)
    ETA[1] := FIND_ETA( RPL[2], RPL[3], TAU[1] );
    ETA[2] := FIND_ETA( RPL[1], RPL[3], TAU[2] );
    ETA[3] := FIND_ETA( RPL[1], RPL[2], TAU[3] );

    (* improvement of the sector/triangle ratios *)
    N[1] := ( TAU[1]/ETA[1] ) / (TAU[2]/ETA[2]);
    N[3] := ( TAU[3]/ETA[3] ) / (TAU[2]/ETA[2]);
    WRITELN('  rho',' ':16,RHO[1]:12:8,RHO[2]:12:8,RHO[3]:12:8);

  UNTIL ( ABS(RHO[2]-RHOOLD) < EPS_RHO );

  WRITELN; WRITELN(' Heliocentric distances [AU]:'); WRITELN;
  WRITELN('  r  ',' ':16,
          NORM(RPL[1]):12:8,NORM(RPL[2]):12:8,NORM(RPL[3]):12:8);
  WRITELN; WRITELN;

  (* derive orbital elements from first and third observation *)
```

```
    ELEMENT (JD[1],JD[3],RPL[1],RPL[3], TP,Q,ECC,INC,LAN,AOP);

  END;
(*-----------------------------------------------------------------------*)

BEGIN

  START(HEADER,RSUN,E,JDO,TEQX);

  GAUSS(RSUN,E,JDO,TP,Q,ECC,INC,LAN,AOP);

  DUMPELEM(TP,Q,ECC,INC,LAN,AOP,TEQX);
  SAVEELEM(TP,Q,ECC,INC,LAN,AOP,TEQX,HEADER);

  (* check solution  *)

  WRITELN;
  IF (DOT(E[2],RSUN[2])>0) THEN
    WRITELN (' Warning: observation in hemisphere of conjunction;',
             '  possible second solution');
  IF (ECC>1.1) THEN
    WRITELN (' Warning: probably not a realistic solution (e>1.1) ');
  IF ( (ABS(Q-0.985)<0.1) AND (ABS(ECC-0.015)<0.05) ) THEN
    WRITELN (' Warning: probably Earth''s orbit solution');

END.
(*-----------------------------------------------------------------------*)
```

APPENDIX G

Example ORBDET Input File

```
1035 Amata: Dan Burtz's Data, Test Case 1
1998 01 21 05.799  03 44 50.43  42 12 41.60  ! Three observations
1998 02 13 01.646  03 53 21.41  40 32 49.20  ! Format: Date (ymdh),
1998 03 13 02.213  04 18 34.54  39 20 22.90  ! RA (hms), Dec (hms)
2000.0                                       ! Equinox
2000.0                                       ! Required equinox
```

APPENDIX H

ORBDET Output Data

ORBDET: orbit determination from three observations
version 93/07/01
(c) 1993 Thomas Pfleger, Oliver Montenbruck

Summary of orbit determination
1035 Amata: Dan Burtz's Data, Test Case 1

Initial data (ecliptic geocentric coordinates (in deg))

Julian Date	2450834.74	2450857.57	2450885.59
Solar longitude	300.98	324.15	352.30
Planet/Comet Longitude	63.66	64.91	69.66
Planet/Comet Latitude	21.81	19.83	17.67

Iteration of the geocentric distances rho [AU]

rho	2.75015994	3.07983981	3.53209164
rho	2.68268840	3.01101420	3.44426096
rho	2.67800252	3.00622284	3.43816414
rho	2.67766167	3.00587426	3.43772070
rho	2.67763586	3.00584790	3.43768743
rho	2.67763398	3.00584598	3.43768499
rho	2.67763384	3.00584584	3.43768481
rho	2.67763383	3.00584583	3.43768480
rho	2.67763383	3.00584582	3.43768480

Heliocentric distances [AU]:

r	3.28296352	3.32420416	3.37302007

Orbital elements (Equinox J 2000.00)

Perihelion date	tp	1996/ 8/21 13.0623h (JD 2450317.04)
Perihelion distance	q[AU]	2.502659
Semi-major axis	a[AU]	3.139193
Eccentricity	e	0.202770
Inclination	i	18.0870 degrees
Ascending node	Omega	2.1808 degrees
Long. of perihelion	pi	325.4512 degrees
Arg. of perihelion	omega	323.2704 degrees
Mean motion	n	0.177205 degrees/day
Mean anomaly	M	85.670976 degrees
Orbital period	P	5.5621 years

ORBDET: orbit determination from three observations
version 93/07/01
(c) 1993 Thomas Pfleger, Oliver Montenbruck

Summary of orbit determination
1035 Amata: Dan Burtz's Data, Test Case 2

Initial data (ecliptic geocentric coordinates (in deg))

Julian Date	2450835.72	2450857.57	2450885.59
Solar longitude	301.98	324.15	352.31
Planet/Comet Longitude	63.66	64.91	69.66
Planet/Comet Latitude	21.73	19.83	17.67

Iteration of the geocentric distances rho [AU]

rho	2.76360006	3.08034154	3.53227255
rho	2.69595418	3.01130373	3.44444420
rho	2.69126012	3.00650219	3.43835253
rho	2.69091902	3.00615324	3.43790990
rho	2.69089249	3.00612615	3.43787602
rho	2.69089057	3.00612418	3.43787353
rho	2.69089043	3.00612404	3.43787335
rho	2.69089042	3.00612403	3.43787333
rho	2.69089042	3.00612403	3.43787333

Heliocentric distances [AU]:

r	3.28504090	3.32443219	3.37316824

Orbital elements (Equinox J 2000.00)

Perihelion date	tp	1996/ 8/21 8.8622h (JD 2450316.87)
Perihelion distance	q[AU]	2.504413
Semi-major axis	a[AU]	3.139943
Eccentricity	e	0.202402
Inclination	i	18.0866 degrees
Ascending node	Omega	2.1676 degrees
Long. of perihelion	pi	325.4894 degrees
Arg. of perihelion	omega	323.3218 degrees
Mean motion	n	0.177142 degrees/day
Mean anomaly	M	85.671276 degrees
Orbital period	P	5.5640 years

ORBDET: orbit determination from three observations
version 93/07/01
(c) 1993 Thomas Pfleger, Oliver Montenbruck

Summary of orbit determination
1035 Amata: Dan Burtz's Data, Test Case 3

Initial data (ecliptic geocentric coordinates (in deg))

Julian Date	2450834.72	2450857.59	2450885.60
Solar longitude	300.96	324.17	352.31
Planet/Comet Longitude	63.71	64.92	69.66
Planet/Comet Latitude	21.29	19.82	17.67

Iteration of the geocentric distances rho [AU]

rho	4.19643404	4.05778655	3.93701682
rho	4.14404303	4.01164478	3.88739874
rho	4.14216131	4.00998664	3.88561449
rho	4.14209142	4.00992501	3.88554832
rho	4.14208882	4.00992272	3.88554586
rho	4.14208873	4.00992264	3.88554577
rho	4.14208872	4.00992264	3.88554577

Heliocentric distances [AU]:

r	4.71512193	4.29416049	3.80401671

Orbital elements (Equinox J 2000.00)

Perihelion date	tp	1998/ 7/18 3.0726h (JD 2451012.63)
Perihelion distance	q[AU]	2.489331
Semi-major axis	a[AU]	-0.733254
Eccentricity	e	4.394909
Inclination	i	18.6437 degrees
Ascending node	Omega	340.4056 degrees
Long. of perihelion	pi	499.3132 degrees
Arg. of perihelion	omega	158.9076 degrees
Mean motion	n	1.569720 degrees/day
Mean anomaly	M	-332.981702 degrees
Orbital period	P	0.6279 years

Warning: probably not a realistic solution (e>1.1)

ORBDET: orbit determination from three observations
version 93/07/01
(c) 1993 Thomas Pfleger, Oliver Montenbruck

Summary of orbit determination
1035 Amata: Dan Burtz's Data, Test Case 4

Initial data (ecliptic geocentric coordinates (in deg))

Julian Date	2450840.78	2450857.59	2450885.63
Solar longitude	307.13	324.17	352.35
Planet/Comet Longitude	63.72	64.92	69.67
Planet/Comet Latitude	21.29	19.82	17.67

Iteration of the geocentric distances rho [AU]

rho	2.83399704	3.08206213	3.53318200
rho	2.76532224	3.01193729	3.44535132
rho	2.76057158	3.00707819	3.43927728
rho	2.76022749	3.00672620	3.43883734
rho	2.76020110	3.00669924	3.43880400
rho	2.76019919	3.00669728	3.43880155
rho	2.76019905	3.00669714	3.43880138
rho	2.76019904	3.00669713	3.43880136
rho	2.76019904	3.00669713	3.43880136

Heliocentric distances [AU]:

r	3.29445640	3.32471097	3.37356737

Orbital elements (Equinox J 2000.00)

Perihelion date	tp	1996/ 8/21 17.4846h (JD 2450317.23)
Perihelion distance	q[AU]	2.504159
Semi-major axis	a[AU]	3.140728
Eccentricity	e	0.202682
Inclination	i	18.0868 degrees
Ascending node	Omega	2.1664 degrees
Long. of perihelion	pi	325.5467 degrees
Arg. of perihelion	omega	323.3803 degrees
Mean motion	n	0.177076 degrees/day
Mean anomaly	M	85.575542 degrees
Orbital period	P	5.5661 years

ORBDET: orbit determination from three observations
version 93/07/01

Summary of orbit determination
1035 Amata: Dan Burtz's Data, Test Case 5

Initial data (ecliptic geocentric coordinates (in deg))

Julian Date	2450841.71	2450857.62	2450885.64
Solar longitude	308.07	324.20	352.35
Planet/Comet Longitude	63.74	64.92	69.67
Planet/Comet Latitude	21.20	19.82	17.67

Iteration of the geocentric distances rho [AU]

rho	2.84730595	3.08268194	3.53300119
rho	2.77840944	3.01234947	3.44518341
rho	2.77364447	3.00747753	3.43911142
rho	2.77329804	3.00712332	3.43867036
rho	2.77327278	3.00709749	3.43863820
rho	2.77327093	3.00709560	3.43863585
rho	2.77327080	3.00709546	3.43863568
rho	2.77327079	3.00709545	3.43863567
rho	2.77327079	3.00709545	3.43863567

Heliocentric distances [AU]:

r	3.29617835	3.32469276	3.37333148

Orbital elements (Equinox J 2000.00)

Perihelion date	tp	1996/ 8/21 4.9719h (JD 2450316.71)
Perihelion distance	q[AU]	2.506152
Semi-major axis	a[AU]	3.140653
Eccentricity	e	0.202028
Inclination	i	18.0864 degrees
Ascending node	Omega	2.1577 degrees
Long. of perihelion	pi	325.5288 degrees
Arg. of perihelion	omega	323.3711 degrees
Mean motion	n	0.177082 degrees/day
Mean anomaly	M	85.670941 degrees
Orbital period	P	5.5659 years

ORBDET: orbit determination from three observations
version 93/07/01

Summary of orbit determination
1035 Amata: Dan Burtz's Data, Test Case 6

Initial data (ecliptic geocentric coordinates (in deg))

Julian Date	2450841.77	2450857.63	2450885.64
Solar longitude	308.13	324.21	352.35
Planet/Comet Longitude	63.75	64.92	69.67
Planet/Comet Latitude	21.20	19.82	17.67

Iteration of the geocentric distances rho [AU]

rho	2.84852169	3.08313344	3.53342489
rho	2.77962613	3.01280417	3.44562781
rho	2.77486068	3.00793197	3.43955718
rho	2.77451574	3.00757926	3.43911772
rho	2.77449272	3.00755568	3.43908782
rho	2.77449102	3.00755394	3.43908565
rho	2.77449090	3.00755382	3.43908550
rho	2.77449089	3.00755381	3.43908549

Heliocentric distances [AU]:

r	3.29668444	3.32509755	3.37373549

Orbital elements (Equinox J 2000.00)

Perihelion date	tp	1996/ 8/21 7.9103h (JD 2450316.83)
Perihelion distance	q[AU]	2.507582
Semi-major axis	a[AU]	3.141972
Eccentricity	e	0.201908
Inclination	i	18.0861 degrees
Ascending node	Omega	2.1416 degrees
Long. of perihelion	pi	325.6097 degrees
Arg. of perihelion	omega	323.4682 degrees
Mean motion	n	0.176970 degrees/day
Mean anomaly	M	85.595332 degrees
Orbital period	P	5.5694 years

APPENDIX I

FIND_ORB Input File

```
J98X01X  C1998 01 21.24164 03 44 50.43 +42 12 41.6        15.9 V    712
J98X01X  C1998 01 21.24550 03 44 50.41 +42 12 40.7        15.9 V    712
J98X01X  C1998 01 21.27009 03 44 50.55 +42 12 32.5        15.9 V    712
J98X01X  C1998 01 21.27427 03 44 50.54 +42 12 31.2        16.0 V    712
J98X01X  C1998 01 22.21771 03 44 55.41 +42 07 42.6        15.9 V    712
J98X01X  C1998 01 22.22047 03 44 55.42 +42 07 41.2        15.9 V    712
J98X01X  C1998 01 22.27830 03 44 55.73 +42 07 23.9        16.1 V    712
J98X01X  C1998 01 22.28038 03 44 55.74 +42 07 23.1        16.1 V    712
J98X01X  C1998 01 27.21590 03 45 46.10 +41 43 02.5        16.5 V    712
J98X01X  C1998 01 27.21848 03 45 46.10 +41 43 01.6        16.6 V    712
J98X01X  C1998 01 27.25006 03 45 46.54 +41 42 53.6        15.7 V    712
J98X01X  C1998 01 27.25433 03 45 46.62 +41 42 51.9        16.0 V    712
J98X01X  C1998 01 27.27655 03 45 46.94 +41 42 45.1        16.0 V    712
J98X01X  C1998 01 27.28177 03 45 47.03 +41 42 43.5        16.0 V    712
J98X01X  C1998 01 28.21384 03 46 01.17 +41 38 18.7        16.1 V    712
J98X01X  C1998 01 28.21614 03 46 01.20 +41 38 18.3        15.9 V    712
J98X01X  C1998 01 28.24293 03 46 01.63 +41 38 10.2        16.0 V    712
J98X01X  C1998 01 28.24633 03 46 01.68 +41 38 09.1        16.1 V    712
J98X01X  C1998 01 28.26623 03 46 01.96 +41 38 03.8        16.0 V    712
J98X01X  C1998 01 28.27069 03 46 02.02 +41 38 02.5        16.0 V    712
J98X01X  C1998 02 13.06861 03 53 21.41 +40 32 49.2        16.6 V    712
J98X01X  C1998 02 13.07160 03 53 21.53 +40 32 48.7        16.5 V    712
J98X01X  C1998 02 13.09028 03 53 22.22 +40 32 44.8        16.4 V    712
J98X01X  C1998 02 13.09264 03 53 22.33 +40 32 44.0        16.4 V    712
J98X01X  C1998 02 13.12376 03 53 23.49 +40 32 37.6        16.4 V    712
J98X01X  C1998 02 13.12638 03 53 23.59 +40 32 37.1        16.4 V    712
J98X01X  C1998 03 13.09222 04 18 34.54 +39 20 22.9        16.2 V    712
J98X01X  C1998 03 13.09472 04 18 34.71 +39 20 22.4        16.4 V    712
J98X01X  C1998 03 13.09684 04 18 34.84 +39 20 22.2        16.1 V    712
J98X01X  C1998 03 13.13267 04 18 37.23 +39 20 18.7        16.2 V    712
J98X01X  C1998 03 13.13863 04 18 37.63 +39 20 17.6        16.3 V    712
J98X01X  C1998 03 13.14065 04 18 37.76 +39 20 17.3        16.1 V    712
```

APPENDIX J

FIND_ORB Output File

```
Orbital elements:
1998XX1
  Perihelion 1996 Aug 21.102725 TT
Epoch 1997 Dec 18.0 TT = JDT 2450800.5
M 85.82430            (2000.0)          P              Q
n 0.17736059    Peri. 323.13457    0.82135348     0.57029497
a 3.1373621     Node    2.20044   -0.42057277     0.61957291
e 0.2025591     Incl.  18.08732   -0.38535309     0.53934502
P 5.56        H 10.4         G 0.15      q 2.5018609
From 32 observations 1998 Jan. 21-Mar. 13;  RMS error 0.360 arcseconds
```

APPENDIX K

Example Astrometry Submission to MPC

USAF Academy Observatory

1998 January 26

Observatory Code: 712
Longitude: 104.8811øW p cos í': +0.77837
Latitude: 39.0070øN p sin í': +0.62625
Height: 2180.0m

41-cm reflector + CCD, GSC reference stars (J2000.0).

Postal adress:

Wetterer
Department of Physics
USAF Academy, Colorado
USA

Object	Date	U.T.	R.A. (2000.0)	Decl.	Mag.	Observer	Comp Stars
Karayusuf	1998 01	24.41848	10 13 41.85	+35 13 28.8	16.1V	Wetterer	8
Karayusuf	1998 01	24.42039	10 13 41.75	+35 13 32.6	15.9V	Wetterer	8
Karayusuf	1998 01	24.43358	10 13 41.27	+35 13 53.9	15.9V	Wetterer	8
Karayusuf	1998 01	24.44428	10 13 40.99	+35 14 09.5	15.8V	Wetterer	8
Amata	1998 01	27.21590	03 45 46.10	+41 43 02.5	16.5V	Wetterer/Burtz	9
Amata	1998 01	27.21848	03 45 46.10	+41 43 01.6	16.6V	Wetterer/Burtz	9
Amata	1998 01	27.25006	03 45 46.54	+41 42 53.6	15.7V	Wetterer/Burtz	9
Amata	1998 01	27.25433	03 45 46.62	+41 42 51.9	16.0V	Wetterer/Burtz	9
Amata	1998 01	27.27655	03 45 46.94	+41 42 45.1	16.0V	Wetterer/Burtz	8
Amata	1998 01	27.28177	03 45 47.03	+41 42 43.5	16.0V	Wetterer/Burtz	8
Amata	1998 01	28.21384	03 46 01.17	+41 38 18.7	16.1V	Burtz/Wetterer	9
Amata	1998 01	28.21614	03 46 01.20	+41 38 18.3	15.9V	Burtz/Wetterer	9
Amata	1998 01	28.24293	03 46 01.63	+41 38 10.2	16.0V	Burtz/Wetterer	9
Amata	1998 01	28.24633	03 46 01.68	+41 38 09.1	16.1V	Burtz/Wetterer	9
Amata	1998 01	28.26623	03 46 01.96	+41 38 03.8	16.0V	Burtz/Wetterer	9
Amata	1998 01	28.27069	03 46 02.02	+41 38 02.5	16.0V	Burtz/Wetterer	9
Ramanujan	1998 01	27.22343	05 26 21.20	+10 25 24.6	17.3V	Wetterer/Burtz	12
Ramanujan	1998 01	27.28451	05 26 20.13	+10 25 40.3	17.7V	Wetterer/Burtz	12
Ramanujan	1998 01	27.28787	05 26 19.97	+10 25 39.8	17.5V	Wetterer/Burtz	12
Ramanujan	1998 01	28.22064	05 26 03.15	+10 28 49.1	17.4V	Burtz/Wetterer	12
Ramanujan	1998 01	28.22256	05 26 03.08	+10 28 49.1	17.4V	Burtz/Wetterer	12
Ramanujan	1998 01	28.24888	05 26 02.53	+10 28 54.3	17.1V	Burtz/Wetterer	12
Ramanujan	1998 01	28.25224	05 26 02.53	+10 28 55.0	17.3V	Burtz/Wetterer	12
Comrie	1998 01	27.22972	10 04 52.35	+19 04 12.7	17.4V	Wetterer/Burtz	10
Comrie	1998 01	27.23221	10 04 52.22	+19 04 14.1	17.1V	Wetterer/Burtz	10
Comrie	1998 01	27.26237	10 04 50.62	+19 04 25.4	17.0V	Wetterer/Burtz	10
Comrie	1998 01	27.26634	10 04 50.43	+19 04 27.4	17.0V	Wetterer/Burtz	10
Comrie	1998 01	28.22870	10 04 01.50	+19 10 32.9	17.1V	Burtz/Wetterer	8
Comrie	1998 01	28.23061	10 04 01.32	+19 10 34.2	17.2V	Burtz/Wetterer	8
Comrie	1998 01	28.25449	10 04 00.03	+19 10 43.8	16.9V	Burtz/Wetterer	8
Comrie	1998 01	28.25843	10 03 59.82	+19 10 45.3	17.0V	Burtz/Wetterer	8

Karayusaf	1998 01 27.23680	10 12 01.87	+36 32 02.6	15.4V	Wetterer/Burtz	9
Karayusaf	1998 01 27.23909	10 12 01.75	+36 32 06.6	15.4V	Wetterer/Burtz	9
Karayusaf	1998 01 27.26940	10 12 00.38	+36 32 57.8	15.4V	Wetterer/Burtz	9
Karayusaf	1998 01 27.27315	10 12 00.28	+36 33 03.1	15.4V	Wetterer/Burtz	9
Karayusaf	1998 01 28.23657	10 11 19.86	+36 59 40.8	16.0V	Burtz/Wetterer	7
Karayusaf	1998 01 28.23846	10 11 19.76	+36 59 43.7	16.0V	Burtz/Wetterer	7
Karauysaf	1998 01 28.26065	10 11 18.71	+37 00 21.0	15.9V	Burtz/Wetterer	7
Karauysaf	1998 01 28.26385	10 11 18.55	+37 00 26.3	16.0V	Burtz/Wetterer	7
Raffinetti	1998 01 27.29867	11 09 48.68	-05 20 32.3	16.6V	Wetterer/Burtz	10
Raffinetti	1998 01 27.30102	11 09 48.59	-05 20 32.0	16.6V	Wetterer/Burtz	10
Raffinetti	1998 01 27.33069	11 09 48.09	-05 20 33.2	16.8V	Wetterer/Burtz	10
Raffinetti	1998 01 27.33454	11 09 48.08	-05 20 32.7	16.9V	Wetterer/Burtz	10
Raffinetti	1998 01 28.29027	11 09 32.38	-05 20 26.6	16.8V	Burtz/Wetterer	10
Raffinetti	1998 01 28.30932	11 09 32.00	-05 20 26.6	16.8V	Burtz/Wetterer	10
Raffinetti	1998 01 28.31381	11 09 31.92	-05 20 26.3	16.9V	Burtz/Wetterer	10
Pien	1998 01 27.30875	11 22 51.58	+13 54 15.8	17.5V	Wetterer/Burtz	6
Pien	1998 01 27.31174	11 22 51.58	+13 54 15.9	17.2V	Wetterer/Burtz	6
Pien	1998 01 27.33708	11 22 50.94	+13 54 26.7	17.5V	Wetterer/Burtz	6
Pien	1998 01 27.34104	11 22 50.67	+13 54 26.1	16.9V	Wetterer/Burtz	6
Pien	1998 01 28.27936	11 22 28.55	+13 59 47.9	16.8V	Burtz/Wetterer	7
Pien	1998 01 28.28119	11 22 28.51	+13 59 48.1	16.9V	Burtz/Wetterer	7
Pien	1998 01 28.30284	11 22 27.92	+13 59 56.3	16.5V	Burtz/Wetterer	7
Pien	1998 01 28.30668	11 22 27.87	+13 59 57.5	16.6V	Burtz/Wetterer	7
Pien	1998 01 28.33435	11 22 27.15	+14 00 07.0	16.5V	Burtz/Wetterer	7
Pien	1998 01 28.32125	11 22 27.51	+14 00 02.2	16.5V	Burtz/Wetterer	7